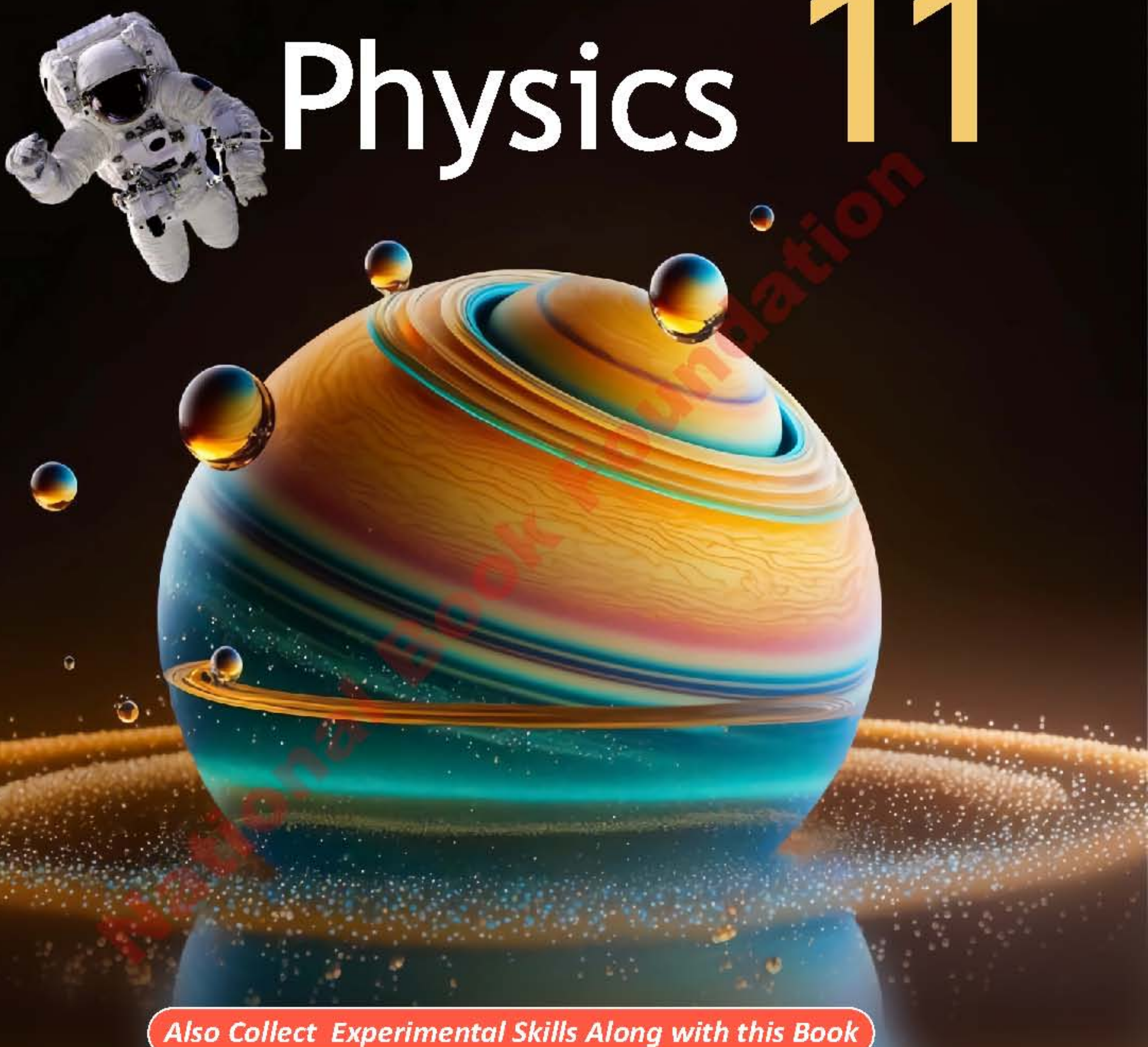
Based on National Curriculum of Pakistan 2022-23

Model Textbook of Physics 11



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Model Textbook of

Physics

Grade

11

National Curriculum Council
Ministry of Federal Education and Professional Training

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Islamabad

Model Textbook of **Physics**
for Grade 11

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Preface

This Model Textbook for Physics Grade 11 has been developed by NBF according to the National Curriculum of Pakistan 2022-2023. The aim of this textbook is to enhance learning abilities through inculcation of logical thinking in learners, and to develop higher order thinking processes by systematically building the foundation of learning from the previous grades. A key emphasis of the present textbook is creating real life linkage of the concepts and methods introduced. This approach was devised with the intent of enabling students to solve daily life problems as they grow up in the learning curve and also to fully grasp the conceptual basis that will be built in subsequent grades.

After amalgamation of the efforts of experts and experienced authors, this book was reviewed and finalized after extensive reviews by professional educationists. Efforts were made to make the contents student friendly and to develop the concepts in interesting ways.

The National Book Foundation is always striving for improvement in the quality of its textbooks. The present textbook features an improved design, better illustration and interesting activities relating to real life to make it attractive for young learners. However, there is always room for improvement, the suggestions and feedback of students, teachers and the community are most welcome for further enriching the subsequent editions of this textbook.

May Allah guide and help us (Ameen).

Dr. Raja Mazhar Hameed
Managing Director

Contents

UNIT 1

MEASUREMENT

Student Learning Outcomes (SLOs)

The students will

- Make reasonable estimates of physical quantities [of those quantities that are discussed in the topics of this grade].
- Express derived units as products or quotients of the SI base units.
- Analyze the homogeneity of physical equations [Through dimensional analysis].
- Derive formulae in simple cases [Through using dimensional analysis].
- Analyze and critique the accuracy and precision of data collected by measuring instruments.
- Assess the uncertainty in a derived quantity [By simple addition of absolute, fractional or percentage uncertainties].
- Justify why all measurements contain some uncertainty.

Physics is based on experimental observations. Observations may be qualitative or quantitative. Qualitative observations have no associated numbers. It deals with facts that can be observed with our five senses: sight, smell, taste, touch and hearing. For instance, colors, shapes and textures of objects are examples of qualitative observations. Observations, like 'water keeps its level' is also an example of qualitative observation. A quantitative observation includes numbers, and is also called a measurement. We can measure mass, time, distance, speed, pressure, force, torque, momentum, and energy. Quantitative observations are more useful to a scientist.

1.1 ESTIMATION OF PHYSICAL QUANTITIES

In our daily life, we may face some situations like: What will be the height of this building? Will the piece of equipment fit in the back of our car or do we need to rent a truck? How long will this download take? About how large a current will be there in this circuit? How many houses could a proposed power plant actually power if it is built? Usually we solve such problems by estimations. In many circumstances, scientists and engineers also need to make estimates of some specific physical quantity with the help of little or no actual data.

Estimation does not mean guessing a formula or a number at random. Let us understand the estimation with the help of a simple example:

Estimation of length: To estimate the height of building, we first count the number of floors it has. Then, estimate the height of a single floor by imagining how many people would have to stand on each other's shoulders to reach the ceiling. In the last, we estimate the height of a person. These estimates give you the height of the building.

An estimation is a rough educated guess to the value of a physical quantity by using prior experience and sound physical reasoning.

An estimation usually includes the identification of correct physical principles and a good guess about the relevant variables. Estimation is very useful in developing a physical sense.

Some of the following kind of strategies may help to improve our skill of estimation:

- **Breaking big things into smaller things or aggregating smaller things into a bigger thing:** When estimating lengths, remember that anything can be a ruler. Thus, imagine breaking a big thing into smaller things, estimate the length of one of the smaller things, and multiply to obtain the length of the big thing. For example, we have estimated the height of a building in the previous paragraphs. Sometimes it also helps to do this in reverse, i.e., to estimate the length of a small thing, imagine a bunch of them making up a bigger thing. For example, to estimate the thickness of a sheet of paper, estimate the thickness of a stack of paper and then divide by the number of pages in the stack. These same strategies of breaking big things into smaller things or aggregating smaller things into a bigger thing can sometimes be used to estimate other physical quantities, such as mass and time. In such situations some of the length, mass and time scales, as shown in table 1.1 may be helpful.

Table 1.1: The estimation of some physical quantities.		
Length (m)	**Mass (kg)**	**Time (s)**
Diameter of proton = 10^{-15}	Mass of electron = 10^{-30}	Mean lifetime of unstable nucleus = 10^{-22}
Diameter of large nucleus = 10^{-14}	Mass of proton = 10^{-27}	Time for single floating-point operating in a supercomputer = 10^{-17}
Diameter of H-tom = 10^{-10}	Mass of bacterium = 10^{-15}	Time period of visible light = 10^{-15}
Diameter of typical virus = 10^{-7}	Mass of mosquito = 10^{-5}	Time period of an atom in solid = 10^{-13}
width of pinky fingernail = 10^{-2}	Mass of hummingbird = 10^{-2}	Time period of nerve impulse = 10^{-3}
Height of 4 year old child = 10^{0}	Mass of 1 liter water = 10^{0}	Time for 1 heartbeat = 10^{0}
length of football ground = 10^{2}	Mass of a Motorcycle= 10^{2}	One day = 10^{5}
Diameter of Earth = 10^{7}	Mass of atmosphere = 10^{19}	One year = 10^{7}
Diameter of solar system = 10^{13}	Mass of Moon = = 10^{22}	Human lifetime = 10^{9}
1 light-year = 10^{16}	Mass of Earth = = 10^{25}	Recorded human history = 10^{11}
Diameter of Milky-Way = 10^{21}	Mass of Sun = = 10^{30}	Age of Earth = 10^{17}
Distance b/w edges of observable universe = 10^{26}	Mass of known universe = 10^{53}	Age of universe = 10^{18}

- **Estimate Areas and Volumes from Lengths:** When dealing with an area or a volume of a complex object, introduce a simple model of the object, such as a sphere or a box. Then, estimate the linear dimensions (such as the radius of the sphere or the length, width, and height of the box) first, and use the estimates to find the volume or area from standard geometric formulas. If you happen to have an estimate of an object's area or volume, you can also do the reverse; that is, use standard geometric formulas to get an estimate of its linear dimensions.
- **Estimate Mass from Volume and Density:** When estimating the mass of an object, it can help first to estimate its volume and then to estimate its mass from estimate of its average density. (recall, density has dimension of mass/length^3, so mass = density × volume). For this, it helps to remember that the density of air is about 1 kg/m^3, the density of water is 10^3 kg/m^3, and the densest everyday solids max out at around 10^4 kg/m^3. Asking yourself whether an object floats or sinks in either air or water gets you a rough estimate of its density. You can also do the reverse: if you have an estimate of an object's mass and its density, you can use them to get an estimate of its volume.

Example 1.1: Estimate the energy required for an adult man to walk up through stairs from ground floor to 1st floor?

Solution:

As, the energy required = $m\,g\,h$

We have to take the following estimations:

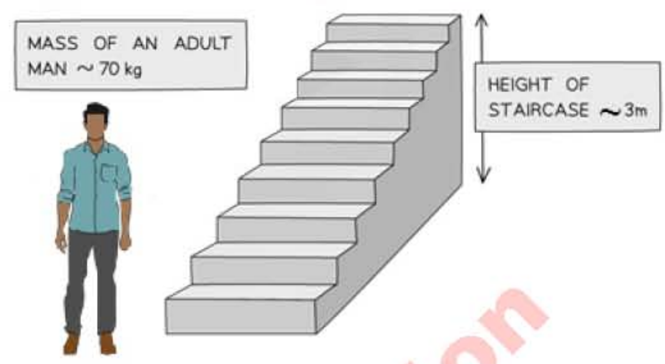
Mass of an adult ≈ 70 kg

Distance between 2 floors ≈ 3 m

So,

Energy required = 70×10×3

= 2100 J



Assignment 1.1

Estimate that how many floating-point operations can a supercomputer do in 1 day?

1.2 DERIVED UNITS IN TERMS OF BASE UNITS

In Grade 9, we have studied about base and derived physical quantities and also their units. We know that derived units can be expressed in terms of base units and are obtained by multiplying or dividing base units with each other. Here we will express some more derived units as products or quotients of the SI base units. Let us first we take force: as,

$$Force = mass \times acceleration$$

$$Force = mass \times \frac{velocity}{time}$$

$$Force = mass \times \frac{displacement}{time^2}$$

Now, we put SI unit for each physical quantity i.e., N for force, kg for mass, m for displacement and s for time, so we get:

$$N = kg \times m/s^2$$

For work we proceed as:

$$Work = Force \times displacement$$

$$Work = mass \times acceleration \times displacement$$

$$Work = mass \times \frac{velocity}{time} \times displacement$$

$$Work = mass \times \frac{displacement}{time^2} \times displacement$$

$$Work = mass \times \frac{(displacement)^2}{(time)^2}$$

Now, we put SI unit for each physical quantity i.e., J for work, kg for mass, m for displacement and s for time, so we get:

$$J = kg \times \frac{m^2}{s^2}$$

Similarly, we can express other derived units as products or quotients of the SI base units. Some examples are shown in the table 1.2.

TABLE 1.2

Name of Derived Quantity	SI Unit	Symbol	In terms of base units
Power	watt	W	$J/s = kg\ m^2/s^3$
Pressure	pascal	Pa	$N\ m^{-2} = kg\ m^{-1}s^{-2}$
Electric Charge	coulomb	C	A s

1.3 DIMENSIONS OF PHYSICAL QUANTITIES

Dimension denotes the qualitative nature of a physical quantity. For example, length, width, height, distance, displacement, radius etc. all are measured in meter because of having same nature so have the same dimensions.

Dimension of a physical quantity is often represented by capital letter enclosed in square brackets [ ]. Dimensions for base quantities are given in the table 1.3.

TABLE 1.3

Sr. No	Physical Quantity	Dimensions
1	mass	[M]
2	length	[L]
3	time	[T]
4	electric current	[I]
5	temperature	[θ]
6	intensity of light	[J]
7	amount of substance	[N]

Dimensions of derived quantities are obtained by multiplication or division of the dimensions of base quantities i.e., from which these quantities are derived. For example, the dimension for area, volume, velocity and acceleration are $[L^2]$, $[L^3]$, $[LT^{-1}]$ and $[LT^{-2}]$ respectively.

Thus, dimensions give the relation of a given physical quantity with base quantities i.e. mass, length, time etc. There are following essential terms which are used in dimensional analysis:

Dimensional Variables: Those physical quantities that have dimensions and have variable magnitude are called dimensional variables. Some dimensional variables are length, velocity, acceleration, force, energy and acceleration etc.

Dimensional Constant: Those physical quantities that have dimensions and constant in magnitude are called dimensional constant. Some examples of dimensional constants are

Planck's constant, gravitational constant, speed of light in vacuum and ideal gas constant etc.

Dimensionless Variables: Those physical quantities that have no dimensions and have variable magnitude are called dimensionless variables. Some examples of dimensionless variables are plane angle, solid angle, strain and coefficient of friction etc.

Dimensionless Constant: Those physical quantities that have no dimensions and constant in magnitude are called dimensionless constant. A pure number (1, 2, 3, ...), the exponential constant (e = 2.718) and π are some examples of dimensionless constant.

1.3.1 Advantages of Dimensions

Using the method of dimensions (called dimensional analysis) we can check the homogeneity of an equation, to derive a possible formula and its units. Dimensional analysis makes use of the fact that dimensions can be treated as algebraic quantities. That is, quantities can be added or subtracted only if they have the same dimensions. Furthermore, the terms on both sides of an equation must have the same dimensions.

(i) The Homogeneity of an Equation

In order to check the correctness of an equation, we have to show that both sides of the equation have the same dimensions, otherwise the equation cannot be considered as physically correct equation. This is called the principle of homogeneity of dimensions.

Let us check whether the equation $v_f = v_i + at$ is dimensionally correct.

$$\text{Dimensions of L.H.S.} = [LT^{-1}]$$

$$\text{Dimensions of R.H.S.} = [LT^{-1}] + [LT^{-2}][T]$$

$$= [LT^{-1}] + [LT^{-1}]$$

$$= 2[LT^{-1}]$$

As, 2 is dimensionless constant, so

Dimensions of L.H.S = Dimensions of R.H.S

Hence, the equation is dimensionally correct.

(ii) To Derive a Possible Formula

Deriving a relation for a physical quantity depends on the correct guessing of various factors on which the physical quantity depends. Let us derive the formula for wavelength of matter waves using dimensional analysis.

As wavelength (λ) of matter waves may depend upon Plank's constant (h), velocity (v) and mass (m) of the particle.

So, the relation for the wavelength (λ) will be of the form:

$$\lambda \propto h^a m^b v^c$$

$$\lambda = (\text{constant})\, h^a m^b v^c \text{ ----------------- } (1)$$

We have to find the values of powers i.e. a, b and c:

Using the dimension on both sides, we get:

$$[L] = \text{constant } [ML^2T^{-1}]^a [M]^b [LT^{-1}]^c$$

OR $$[M^0 L^1 T^0] = \text{constant } [M]^{a+b} [L]^{2a+c} [T]^{-a-c} \text{ ----------------- } (2)$$

Equating the powers of M on both sides of equation (2), we get:

$$a + b = 0 \text{ ----------------- } (3)$$

Equating the powers of L on both sides of equation (2), we get:

$2a + c = 1$ ----------------- (4)

Equating the powers of T on both sides of equation (2), we get:

$-a - c = 0$ ----------------- (5)

On solving (3), (4) and (5), we get:

$a = 1, b = -1$ and $c = -1$

Put the values of a, b, and c in (1), we get:

$\lambda = (\text{constant})\, h^1 m^{-1} v^{-1}$

OR $\lambda = (\text{constant}) \times \dfrac{h}{mv}$

1.3.2 Limitations of Dimensional Analysis

Some limitations of dimensional analysis are:

1) Dimensional analysis does not distinguish between the physical quantities having same dimensions. For example, if the dimensional formula of a physical quantity is $[ML^2T^{-2}]$ it may be work or energy or torque.
2) Dimensional analysis cannot be used to derive a formula containing trigonometric function, exponential functions, logarithmic function, etc.
3) Dimensional analysis cannot determine the dimensionless constant when deriving a possible formula.
4) Dimensional analysis doesn't always prove that a relation is physically correct although relation is dimensionally correct. However, a dimensionally wrong equation is always wrong.

Example 1.2: Derive formula for the time period of simple pendulum using dimensional analysis.

Solution: The time period of the simple pendulum is possibly depending on mass of the bob (m), length of the pendulum (l), angle which the string makes with vertical (θ) and acceleration due to gravity (g). So, the relation for the time period T will be of the form:

$T \propto m^a l^b \theta^c g^d$

$T = (\text{constant})\, m^a l^b \theta^c g^d$ ----------------- (1)

We have to find the values of powers i.e. a, b, c and d:

Using the dimension on both sides, we get:

$[M^0 L^0 T] = \text{constant}\, [M]^a [L]^b [LL^{-1}]^c [LT^{-2}]^d$

$[M^0 L^0 T] = \text{constant}\, [M]^a [L]^{b+d} [T]^{-2d}$ ----------------- (2)

Equating the powers of M on both sides of equation (2), we get:

$a = 0$ ----------------- (3)

Equating the powers of L on both sides of equation (2), we get:

$b + d = 0$ ----------------- (4)

Equating the powers of T on both sides of equation (2), we get:

$-2d = 1$

Or $d = -1/2$ ----------------- (5)

Put $d = -1/2$, in (4), we get:

$b = 1/2$

Put the values of a, b, c and d in (1)

$$T = (\text{constant})\ m^{o}\ l^{1/2}\ g^{-1/2}$$

$$T = (\text{constant}) \times \sqrt{\frac{l}{g}}$$

Where the constant can be found by experiment, which is 2π.

Assignment 1.2

Which of the following relationships is dimensionally consistent with an expression yielding a value for acceleration? In these equations, x is a distance, t is time, and v is velocity.

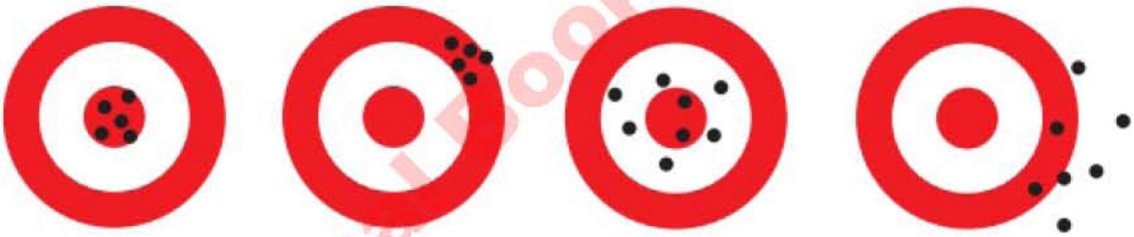
(a) v/t^2 (b) v/x^2 (c) v^2/t (d) v^2/x

1.4 PRECISION AND ACCURACY

Science is built on observation and experiment, i.e., on measurements. Precision is measurements of the same physical quantity agree with each other. Accuracy is measurements of the same physical quantity agree with standard or true value. Accuracy describes how well we eliminate systematic error. Hence:

Precision refers to the closeness of measured values to each other, while accuracy refers to the closeness of a measured value to a standard or true value.

For Your Information



(a) precise and accurate (b) precise but not accurate (c) not precise but accurate (d) neither precise nor accurate

Several independent trials of shooting at a bullseye target illustrate the difference between being accurate and being precise. Accuracy is how close an arrow gets to the bull's-eye center. Precision is how close a second arrow is to the first one (regardless of closeness to the target).

To understand the concept of precision and accuracy, consider that a person weighs exactly 160.0 pounds and he weight himself three times on three different scales. Results of the scales are:

Scale A:	170.1, 169.9 and 170.0 pounds.
Scale B:	161, 162 and 158 pounds.
Scale C:	159.9. 160.0 and 160.1 pounds.

In this case, weight measured by scale A is very precise, but not accurate. Weight measured by scale B is fairly accurate but not precise. Weight measured by scale C is both precise and accurate.

The precision of a measurement is associated with least count of the measuring instruments. Smaller the least count of the measuring instrument greater will be its precision.
Precision is indicated by absolute uncertainty in measurement. Accuracy is indicated by the fractional or percentage uncertainty or error in measurement. Smaller the magnitude of fractional or percentage uncertainty or error, greater will be its accuracy.

1.5 UNCERTAINTIES

In Grade 9, we have studied about the sources of human error, systematic error and random error in experiments. We studied that the difference between the true value and observed value of a measurement is called error. i.e.,

Error = observed value − true value

In a measurement the error may occur due to

- Negligence or inexperience of a person.
- Using a faulty apparatus.
- Inappropriate method or technique.

Error may be divided into the following three types:

- Personal Error
- Systematic Error
- Random Error

Here we will study about uncertainty.

Uncertainty is the range of possible values within which the true value of the measurement lies.

Tip for Solving Numerical Problems Symbolic Solutions!

When solving problems, it is very useful to perform the solution completely in algebraic form and wait until the very end to enter numerical values into the final symbolic expression. This method will save many calculator keystrokes, especially if some quantities cancel so that you never have to enter their values into your calculator! In addition, you will only need to round once, on the final result.

For example, a measurement of 3.06 mm ± 0.02 mm means that the experimenter is confident that the actual value for the quantity being measured lies between 3.04 mm and 3.08 mm. "Uncertainty is a quantitative measurement of variability in the data".
All measurements have a degree of uncertainty. This is caused by two factors, the limitation of the measuring instrument (systematic error) and the skills of the experimenter making the measurements.
Absolute uncertainty is equal to the least count of a measuring instrument, for example the length of a glass slab measured with meter rod is 37.5 cm. The least count of meter rod is 1 mm = 0.1 cm, so the absolute uncertainty in measured value will be ± 0.1 cm i.e., ±0.05 cm uncertainty develops at each end. For example, if one end of the slab coincides with 20.5 cm mark and the other coincides with 58.0 cm mark of meter rule, the length of the slab along with uncertainty is given by

$$(58.0 \pm 0.05)\ \text{cm} - (20.5 \pm 0.05)\ \text{cm} = (37.5 \pm 0.1)\ \text{cm}$$

It means that the length of slab is between 37.4 cm and 37.6 cm.

Remember!

If $x + \Delta x = (2.0 \pm 0.1)$ mm, then Actual/Absolute uncertainty is

$\Delta x = \pm 0.1$ mm

Fractional uncertainty is

$\frac{\Delta x}{x} = 0.05$

Percentage uncertainty is

$\frac{\Delta x}{x} \times 100\% = 5\ \%$

In the above measurement precision is ±0.1 cm, which is equal to the magnitude of absolute uncertainty.
The accuracy in the measurement is the magnitude of fractional error. Here

$$\text{Fractional uncertainty} = \frac{\text{absolute uncertainty}}{\text{measured value}}$$

$$= \frac{\pm 0.1}{37.5} = \pm\ 0.003$$

$$Percentage\ uncertainty = fractional\ uncertainty \times 100\,\%$$

Smaller the magnitude of fractional (relative) uncertainty or error greater will be the accuracy in measurement.

1.5.1 Rules for Calculating Uncertainties in Final Result

There are some rules for calculating uncertainties in different cases but we need to be very careful whether we use the absolute or percentage uncertainty in each case.

Let x and y are two different physical quantities with uncertainties Δx and Δy respectively. If z is a physical quantity which is obtained by operating x and y then the propagated uncertainty Δz in the result can be calculated by using the following rules.

a) Rule for Sum and Difference

If two or more than two measured quantities are added or subtracted, then their absolute uncertainties are added to get uncertainty in the result.

If $z = x + y$ or $z = x - y$

then $\Delta z = \pm (\Delta x + \Delta y)$

For example, if $x \pm \Delta x = (24.0 \pm 0.1)$ cm

and $y \pm \Delta y = (30.0 \pm 0.1)$ cm

then $\Delta z = \pm 0.2$ cm

b) Rule for Multiplication and Division

If two or more than two quantities are multiplied or divided, then their percentage uncertainties are added to get uncertainty in the result.

If $z = xy$ or $z = x/y$,

Then % uncertainty in z = % uncertainty in x + % uncertainty in y

c) Rule for Power of a Quantity

The total uncertainty in power of a quantity is equal to the percentage uncertainty multiplied with that power.

If $z = x^3$,

then percentage uncertainties in z = ± 3 (percentage uncertainty in x)

d) Uncertainties in average values of many measurements

The uncertainty in the average value is calculated by adopting the following steps.

i) Find the average of measured values.
ii) Find the deviation of each value from the average.
iii) The mean deviation is the uncertainty in the average.

For example, three readings are recorded for the radius of a small cylinder as

$r_1 = 1.50$ cm, $r_2 = 1.51$ cm and $r_3 = 1.52$ cm

The uncertainty in the average radius is calculated as

Finding average $\bar{r} = \dfrac{r_1 + r_2 + r_3}{3}$

$$= \frac{1.50\ cm + 1.51\ cm + 1.52\ cm}{3} = 1.51\ cm$$

Finding deviation $\Delta r_1 = \bar{r} - r_1 = 1.51 - 1.50 = 0.01$ cm

$\Delta r_2 = \bar{r} - r_2 = 1.51 - 1.51 = 0$ cm

$\Delta r_3 = \bar{r} - r_3 = 1.51 - 1.52 = 0.01$ cm

Finding mean deviation $< \Delta r > = \dfrac{\Delta r_1 + \Delta r_2 + \Delta r_3}{3}$

$= \dfrac{0.01\,cm + 0\,cm \quad 0.01\,cm}{3}$

= 0.0067 cm = 0.007 cm

e) Uncertainty in Timing Experiment

The time period T of a vibrating body can be found by dividing time of multiple vibrations to the number of vibrations.

$$T = \frac{\textit{Time of multiple vibrations}}{\textit{No. of vibrations}}$$

The uncertainty in time period ΔT is found by dividing least count (L.C) of the time recording device to the number of vibrations.

$$\Delta T = \frac{L.C}{\textit{No. of vibrations}}$$

For example, the time recorded for 20 vibrations of a pendulum is t = 35.2 s. Let the least count of stop watch used is 0.1 s (1/10 s). So, the uncertainty in measured time is (35.2 s ± 0.1 s).

Then the time period of the pendulum is obtained as:

T = 35.2/20 = 1.76 s

Uncertainty in time period = ΔT = 0.1/20 = 0.005 s

So, T + ΔT = (1.76 ± 0.005) s

Example 1.3: If voltage measured across a conductor is V ± ΔV = (7.3 ± 0.1) volts and current is I ± ΔI = (2.73 ± 0.051) A. Find the resistance and uncertainty in it.

Given: V ± ΔV = (7.3 ± 0.1) volts I ± ΔI = (2.73 ± 0.051) A

To Find: R ± ΔR = ?

Solution: According to ohm's law, R is calculated as

R = V/I = 7.3/2.73 = 2.7 Ω

Percentage uncertainty in *V* is

$= \dfrac{\Delta V}{V} \times 100\% = \dfrac{0.1}{7.3} \times 100\,\% = 1.37\,\% = 1\,\%$

Percentage uncertainty in *I* is

$= \dfrac{\Delta I}{I} \times 100\% = \dfrac{0.05}{2.73} \times 100\,\% = 1.83\,\% = 2\,\%$

Thus, the total uncertainty in R is

= 1 % + 2 % = 3 %

So R ± ΔR = 2.7 ± 3 %

$= 2.7\,\Omega \pm \left(\dfrac{3}{100} \times 2.7\right)\,\Omega$

= (2.7 ± 0.08) Ω

Example 1.4: If radius of a circular disc is measured as 2.25 cm with uncertainty ± 0.01 cm. Find its surface area with uncertainty in it.

Given: r = 2.25 cm, Δr = ± 0.01 cm

To Find: $A \pm \Delta A = ?$

Solution: As, $A = \pi r^2 = 3.14 \times 2.25^2 = 15.90\ cm^2$

Percentage uncertainty in r $= \frac{\Delta r}{r} \times 100\ \% = \frac{0.01}{2.25} \times 100\ \% = 0.4\ \%$

Percentage uncertainty in area is $= 2 \times 0.4\ \% = 0.8\ \%$

So, $\Delta A = 0.8\ \% \times 15.90\ cm^2 = 0.13\ cm^2$

Thus $A \pm \Delta A = (15.90 \pm 0.13)\ cm^2$

Assignment 1.3

The radius of a circle is measured to be (10.5 ± 0.2) m. Calculate (a) the area and (b) the circumference of the circle, also give the uncertainty in each value.

SUMMARY

- **Estimation** does not mean guessing a formula or a number at random. An estimation is a rough educated guess to the value of a physical quantity by using prior experience and sound physical reasoning.
- **Derived units** can be expressed in terms of base units and are obtained by multiplying or dividing base units with each other.
- **Dimension** denotes the qualitative nature of a physical quantity.
- In order to check the correctness of an equation, we have to show that both sides of an equation have the same dimensions, otherwise the equation cannot be physically correct. This is called the **principle of homogeneity of dimensions.**
- **Uncertainty** is the range of possible values within which the true value of the measurement lies.
- **Absolute uncertainty** is equal to the least count of a measuring instrument.
- **Precision** refers to the closeness of measured values to each other.
- **Accuracy** refers to the closeness of a measured value to a standard or true value.

EXERCISE

Multiple Choice Questions

Encircle the Correct option.

1) The mean diameter of a wire is found to be (0.50±0.02) mm. The percentage uncertainty in the diameter is:

A. 2%　　B. 4%　　C. 6%　　D. 8%

2) A reaction takes place that is expected to yield 171.9 g of product, but it only yields 154.8 g. What is the percent error for this experiment?

A. 17.1%　　B. 90.1%　　C. 111.0%　　D. 9.9%

3) Three different people weigh a standard mass of 2.00 g on the same balance. Each person obtains a reading of exactly 7.32 g for the mass of the standard. These results imply that the balance that was used is

A. both accurate and precise B. neither accurate nor precise

C. accurate but not precise D. precise but not accurate.

4) Dimension of universal gravitational constant is

A. $[M^{-2}L^{3}T^{-2}]$ B. $[M^{3}L^{-1}T^{-2}]$ C. $[M^{-1}L^{3}T^{-2}]$ D. $[M^{-3}L^{3}T^{-2}]$

5) A measurement which on repetition gives same or nearly same result is called

A. Accurate B. average C. precise D. estimated

6) A student is measuring the time of an event by using stopwatch. He takes 5 measurements as: 3.0 s, 3.2 s, 3.4 s, 2.8 s, 3.1 s. What is the uncertainty in the results?

A. ±0.3 s B. ±0.6 s C. ±3.1 s D. ±7.75 s

7) Which of the following quantity has different dimension?

A. Force B. weight C. Modulus of elasticity D. Tension

8) If the dimensions of a physical quantity are given by $[L^{a} M^{b} T^{-c}]$, then the physical quantity will be

A. force, if a = -1, b = 0, c = -2 B. pressure, if a = -1, b = 1, c = -2

C. velocity, if a = 1, b = 0, c = 1 D. acceleration, if a = 1, b = 1, c = -2

9) Order of magnitude of $(10^{6} + 10^{3})$ is

A. 10^{18} B. 10^{9} C. 10^{6} D. 10^{3}

10) Which of the following may be used as a valid formula to calculate speed of ocean waves? [v = speed, g = acceleration due to gravity, λ = wavelength, ρ=density, h= depth].

A. $v = \sqrt{\lambda g}$ C. v = ρgh C. v = gh/λ D. v = λgh

Short Questions

1) Create a table to show reasonable estimate of some physical quantities.

2) Express the units of the following derived quantities in term of base units. (a) Force (b) Work (c) Power (d) Pressure (e) Electric charge.

3) Why is it important to use an instrument of smallest resolution?

4) What is the importance of increasing the number of readings in an experiment?

5) What is the difference between precision and accuracy?

6) What is the principle of homogeneity of dimensions?

7) A ball is thrown in the air and 5 different students are individually measuring the time it takes to fall back down using stopwatches. The times obtained by each student are the following: 6.2 s, 6.0 s, 6.4 s, 6.1 s, 5.8 s. (i) What is the uncertainty of the results? (ii) How should the resulting time be expressed?

8) The energy of a photon is given by $E = hf$, find the dimensions of Plank's constant h, where f is frequency.

Comprehensive Questions

1) Define and explain the term uncertainty.
2) Discuss the rules for calculating uncertainty propagation in the final results in different cases.
3) What does the dimension of a physical quantity mean? What are its advantages, explain with the help of examples?
4) What is meant by estimation of a physical quantity? Explain with the help of an example.

Numerical Problems

(1) Estimate number of heartbeats in a lifetime?

(2) Determine the dimensions of each of the following quantities.

a) $\frac{v^2}{a\,x}$ b) $\frac{a\,t^2}{2}$

(Ans: No, [L])

(3) If A = $\frac{X^2}{Y^2\,Z}$ then find the percentage uncertainty in A. The percentage uncertainties in X, Y and Z are 1 %, 1 % and 2 % respectively.

(Ans: 6 %)

(4) A spherical ball of radius *r* experiences a resistive force *F* due to the air as it moves through the air at speed v. The resistive force F is given by the expression

$$F = c\,r\,v$$

Where c is constant. By using dimensions, derive the SI base unit of the constant c.

(Ans: kg $m^{-1}s^{-1}$)

(5) The pressure (P) at a depth (h) in an incompressible fluid of density (ρ) is given by

$$P = \rho\,g\,h$$

Where g is acceleration due to gravity. Check the homogeneity of this equation.

(6) Estimate that how many protons are there in a bacterium? **(Ans: 10^{12} protons)**

(7) Estimate that how many hydrogen atoms does it take to stretch across the diameter of the Sun? **(Ans: 10^{19} hydrogen atoms)**

(8) The current passing through a resistor R = (13 ± 0.5) Ω is I = (3 ± 0.1) A.

a) Calculate the power consumed (correct to one significant figure).

b) Find the percentage uncertainty of the current passing through the resistor.

d) Find the percentage uncertainty of the resistance.

c) Find the absolute uncertainty of the electrical power.

(Ans: 117 W, 3 %, 3.84 %, 11.7 W)

VECTORS

UNIT 2

Student Learning Outcomes (SLOs)

The students will

- **Represent a vector in 2-D as two perpendicular components.**
- **Describe the product of two vectors (dot and cross-product) along with their properties.**

In our daily life we used to deal with physical quantities, such as mass, time, length, velocity, acceleration, force, work and torque etc. Some physical quantities, known as scalar and can be completely understood by a number and a proper unit only. On other hand, some physical quantities require direction along with a number and a unit, such quantities are known as vectors. Displacement, force, acceleration, torque and electric flux etc., are the examples of vector quantities. Vectors are very useful in our daily life. For example, a signboard provides information about distances and directions to other locations relative to the location of the signboard. Distance is a scalar quantity. Knowing the distance alone is not enough to get to the location. We must also know the direction from the signboard to the destination. The direction along with the distance, is a vector quantity called the displacement. Therefore, signboards give information about displacement from the signboard to the destination.

Graphically a vector can be completely expressed by a line with an arrow head. Length of the line represents the magnitude according to the proper scale and the arrow head represents the direction. We have studied about the addition of vectors in grade 9, here we shall study about the components of vectors and about the products of vectors.

2.1 RECTANGULAR COMPONENTS OF A VECTOR

Components are the effective parts of a vector in different directions. In 2-D, we take x-component and y-component. The component along x-axis is called horizontal or x-component and the component along y-axis is called vertical or y-component. Both the x-component and y-components are perpendicular to each other and are called rectangular components.

The components of a vector which are mutually perpendicular are called rectangular components.

Consider a vector 'A' making an angle 'θ' with the horizontal, as shown in the figure 2.1. Perpendiculars are drawn from the head of vector **A** on x-axis and y-axis. The projection OQ of vector **A** along x-axis is x-component of vector **A** and is represented by $\mathbf{A_x}$. The projection OS of vector **A** along y-axis is y-component of vector **A** and is represented by $\mathbf{A_y}$. So, the vector **A** in its component form can be written as:

$$\mathbf{A} = \mathbf{A_x} + \mathbf{A_y} \qquad \text{\_\_\_\_\_\_ (2.1)}$$

OR

$$\mathbf{A} = A_x\,\hat{i} + A_y\,\hat{j} \qquad \text{\_\_\_\_\_\_ (2.2)}$$

Here, $\hat{i}$ and $\hat{j}$ are representing directions along +x-axis and +y-axis, respectively, and are called unit vectors.

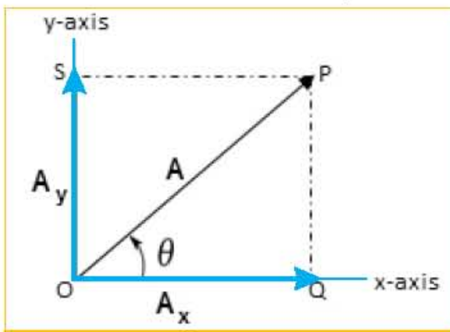


Figure 2.1: Rectangular components of a vector.

2.1.1 Rectangular Components of a Vector:

In order to find the magnitude of rectangular components A_x and A_y of a vector **A**, we apply the trigonometric ratios. For this, consider the triangle OPQ, as shown in figure 2.1. As:

$$\sin\theta = \frac{perpendicular}{hypotenuse}$$

OR $\sin\theta = \frac{A_y}{A}$

OR $A_y = A \sin\theta$ ______ (2.3)

Similarly,

$$\cos\theta = \frac{base}{hypotenuse}$$

OR $\cos\theta = \frac{A_x}{A}$

OR $A_x = A \cos\theta$ ______ (2.4)

From equation (2.3) and (2.4), we can find magnitude of perpendicular components A_x and A_y.

2.1.2 Finding a Vector from its Components:

If the rectangular components A_x and A_y of a vector **A** are given, then magnitude 'A' and its direction 'θ' can be found. Again we consider the triangle OPQ, as shown in figure 2.1. By applying Pythagoras theorem, we get:

$$(OP)^2 = (OQ)^2 + (QP)^2 \text{ ______ (i)}$$

As QP = OS, so equation (i) can be written as:

$$(OP)^2 = (OQ)^2 + (OS)^2 \text{ ______ (ii)}$$

From the figure, OP = A, OQ = A_x and OS = A_y, hence equation (ii) becomes:

$$A^2 = (A_x)^2 + (A_y)^2$$

OR $A = \sqrt{(A_x)^2 + (A_y)^2}$ ______ (2.5)

For finding the direction angle 'θ' of a vector, we apply trigonometric ratio of tangent on triangle OPQ, as:

$$\tan\theta = \frac{perpendicular}{base}$$

OR $\tan\theta = \frac{A_y}{A_x}$

OR $\theta = \tan^{-1}\frac{A_y}{A_x}$ ______ (2.6)

By using equations (2.5) and (2.6) , we can find the magnitude and direction of a vector.

Example 2.1: A peddler is pushing a trolley on a horizontal road with a force of 50 N making 30° with the road. Find the horizontal and vertical components of the force.

Given: Magnitude of force = F = 50 N

Angle of the force with horizontal = θ = 30°

To Find: F_x = ? F_y = ?

Solution: To find x-component of the force F, we use the equation,

$$F_x = F\cos\theta$$

Putting values, we get:

$$F_x = 50 \cos 30^\circ$$

OR $$F_x = 43.3\ N$$

To find y-component of the force F, we use the equation,

$$\boxed{F_y = F \sin\theta}$$

Putting values, we get:

$$F_y = 50 \sin 30^\circ$$

$$\boxed{F_y = 25\ N}$$

Assignment 2.1

Fatima is pulling her trolley bag while climbing up the ramp at her school gate. Find the force with which she is pulling her bag, if x-component and y-component of her force are 12 N and 5 N, respectively.

2.2 PRODUCT OF VECTORS

When two vector quantities are multiplied then the product may be a scalar quantity or a vector quantity. The product obtained depends upon the nature of given vectors. Let us discuss the two types of product in the following sections.

2.2.1 Scalar Product or Dot Product:

When the product of two vector quantities gives a scalar quantity then the product is called scalar product.

Scalar product of two vectors is represented by putting a dot (·) between the symbol of the two vectors, therefore it is also known as dot product.

Let us take two vectors **A** and **B** that are making an angle θ with each other, as shown in the figure 2.2. The dot product between these two vectors can be denoted by **A.B** and is defined as:

$$\mathbf{A} \cdot \mathbf{B} = A\,B \cos\theta \qquad \text{\_\_\_\_\_\_ (2.6)}$$

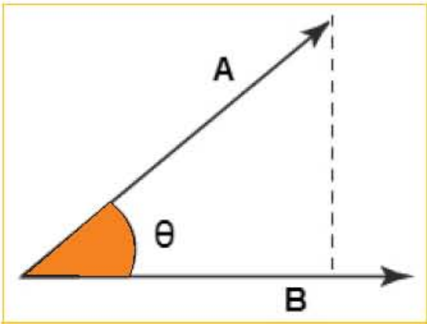


Figure 2.2: Two vectors **A** and **B** that are making an angle θ with each other.

Here, A and B are the magnitude of the vectors **A** and **B**. Thus, the scalar product of two vectors is obtained by multiplying their magnitudes with the cosine of the angle between them.

Equation (2.6) can also be written as:

$$\mathbf{A} \cdot \mathbf{B} = (A \cos\theta)\, B \qquad \text{\_\_\_\_\_\_ (i)}$$

It is shown in the figure 2.3 that (A cos θ) is the component of vector **A** in the direction of vector **B**. So, equation (i) can also be written as:

$$\mathbf{A} \cdot \mathbf{B} = (\text{component of vector } \mathbf{A} \text{ along } \mathbf{B})\, B \qquad \text{\_\_\_\_\_\_ (ii)}$$

Similarly, it can be shown that

A · B = A (component of vector **B** along **A**) ______ (iii)

Hence, from equation (ii) and (iii), we can also define scalar product as:

The product of magnitude of either vector with the component of other vector in the direction of first vector.

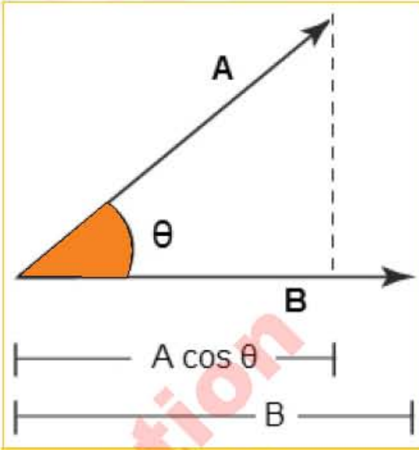


Figure 2.3

Examples of Scalar Product:

- **Work:** Work (W) is an example of scalar product. Work is a scalar quantity which is the scalar product of displacement **d** and force **F**. Mathematically, it can be written as:

$$W = \mathbf{F} \cdot \mathbf{d}$$

- **Power:** Power (P) is also an example of scalar product. It is equal to the scalar product of force **F** and velocity **v**. Mathematically, it can be written as:

$$P = \mathbf{F} \cdot \mathbf{v}$$

- **Electric Flux:** Electric flux Φ a scalar quantity which is the scalar product of electric field intensity **E** and vector area **A**. Mathematically, it can be written as:

$$\Phi = \mathbf{E} \cdot \mathbf{A}$$

Properties of Scalar Product:

1) Scalar product of two vectors obey the commutative property, i.e.,

$$\mathbf{A} \cdot \mathbf{B} = A\,B\cos\theta$$

$$= B\,A\cos\theta = \mathbf{B} \cdot \mathbf{A}$$

Hence, $\mathbf{A} \cdot \mathbf{B} = \mathbf{B} \cdot \mathbf{A}$

2) Scalar product of Two orthogonal (perpendicular) vectors is equal to zero, i.e.,

$$\mathbf{A} \cdot \mathbf{B} = AB\cos 90^\circ = 0$$

Similarly, for mutually perpendicular unit vectors, we can show:

$$\hat{\imath} \cdot \hat{\jmath} = \hat{\jmath} \cdot \hat{k} = \hat{k} \cdot \hat{\imath} = 0$$

Note that the dot product of a vector with the null vector is also zero.

3) Scalar product of two parallel vectors has maximum value, and is equal to the product of their magnitudes only. Hence:

$$\mathbf{A} \cdot \mathbf{B} = A\,B\cos 0^\circ = AB$$

Scalar product of a vector with itself is equal to the square of its magnitude, i.e.,

$$\mathbf{A} \cdot \mathbf{A} = A\,A\cos 0^\circ = A^2$$

Similarly, we can show that the dot product of a unit vector with itself is unity, i.e.,

$$\hat{\imath} \cdot \hat{\imath} = \hat{\jmath} \cdot \hat{\jmath} = \hat{k} \cdot \hat{k} = 1$$

4) Scalar product of two antiparallel vectors is negative, i.e.,

$$A \cdot B = AB \cos 180^o = -AB$$

5) Scalar product of two vectors in terms of their rectangular components can be given as:

$$A \cdot B = (A_x \hat{i} + A_y \hat{j} + A_z \hat{k}) \cdot (B_x \hat{i} + B_y \hat{j} + B_z \hat{k})$$

$$A B \cos\theta = A_x B_x + A_y B_y + A_z B_z$$

$$\cos\theta = \frac{\vec{A} \bullet \vec{B}}{AB} \quad \text{\_\_\_\_\_\_ (2.7)}$$

OR

$$\cos\theta = \frac{A_x B_x + A_y B_y + A_z B_z}{AB} \quad \text{\_\_\_\_\_\_ (2.8)}$$

Example 2.2: Find the power delivered by the engine for attaining velocity (3, 4) m/s, while it exerts a force (8, -2) N.

Given: Velocity = **v** = $(3\hat{i} + 4\hat{j})$ m/s Force = **F** = $(8\hat{i} - 2\hat{j})$ N

To Find: Power Delivered = P = ?

Solution: As we know that power is the scalar product of force and velocity hence:

$$P = F \cdot v$$

$$P = (3\hat{i} + 4\hat{j}) \cdot (8\hat{i} - 2\hat{j})$$

$$P = 24 - 8 = 16 \text{ W}$$

Assignment 2.2

If vectors A= $5\hat{i} + \hat{j}$ and B = $2\hat{i} + 4\hat{j}$, then find:

(i) projection of A on B. **(ii) projection of B on A.**

2.2.2 Vector Product or Cross Product:

When the product of two vector quantities gives a vector quantity then the product is called vector product.

vector product of two vectors is represented by putting a cross (x) between the symbol of the two vectors, therefore it is also known as cross product.

Let us take two vectors A and B that are making an angle θ with each other, as shown in the figure 2.4. The cross product between these two vectors can be denoted by AxB and is defined as:

$$A \times B = A B \sin\theta \, \hat{n} \quad \text{\_\_\_\_\_\_ (2.9)}$$

Here, A and B are the magnitude of the vectors **A** and **B**. Thus, the vector product of two vectors is obtained by multiplying their magnitudes with the sine of the angle between them.

In equation (2.9), $\hat{n}$ represents direction of the vector product, which is always perpendicular to the plane containing **A** and **B**. The direction of cross product ($\hat{n}$) can be found by right hand rule of vector product, as shown in figure 2.5. It can be stated as:

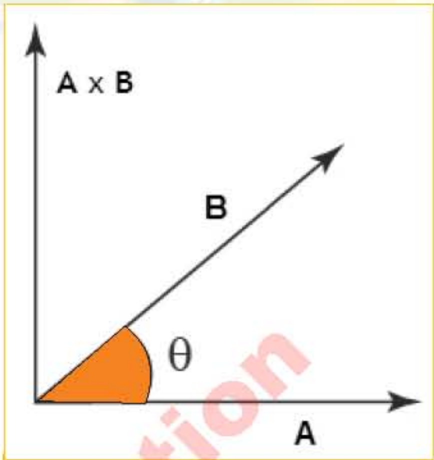


Figure 2.4: Cross product of two vectors **A** and **B** is shown by **AxB**.

Rotate the fingers of your right hand in the direction from first vector to the second vector through smaller angle of the two possible angles with erect thumb. The direction of the product will be along the direction of erect thumb.

Examples of Vector Product:

- Torque (τ) is an example of vector product, in which force (**F**) is multiplied with position vector (**r**).

$$\boldsymbol{\tau} = \mathbf{r} \times \mathbf{F} = r\,F \sin\theta\,\hat{n}$$

- Angular momentum (**L**) is also an example of vector product. It is the cross product of linear momentum **P** and position vector **r**, i.e.,

$$\mathbf{L} = \mathbf{r} \times \mathbf{P} = r\,p \sin\theta\,\hat{n}$$

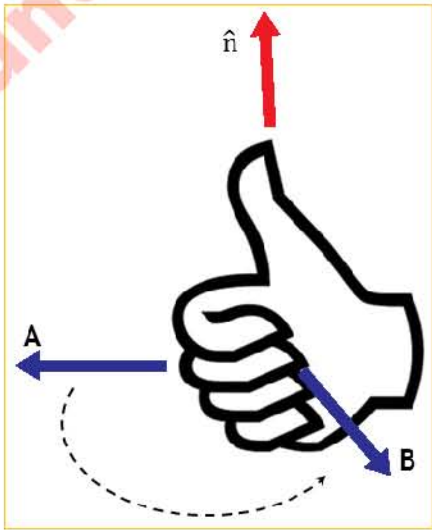


Figure 2.5: Right hand rule.

Properties of Vector Product:

1) Vector product is not commutative but is anti-commutative. It means that by changing the order of vectors, the direction of vector product gets reverse, as shown in figure 2.6. i.e.,

$$\mathbf{A} \times \mathbf{B} \neq \mathbf{B} \times \mathbf{A} \qquad \mathbf{A} \times \mathbf{B} = -\,\mathbf{B} \times \mathbf{A}$$

Here, magnitude of **A x B** is same as **B x A**. According to right hand rule, the direction of **A x B** is pointing upward given by $\hat{n}$ and the direction of **B x A** is pointing downward given by $-\hat{n}$.

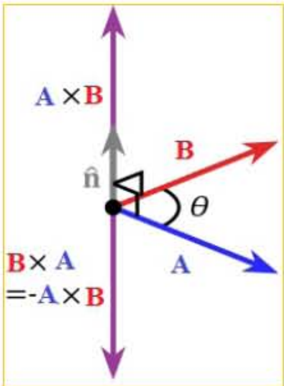


Figure 2.6

2) Vector product of two parallel or anti-parallel vectors is zero. i.e.,

$$\mathbf{A} \times \mathbf{B} = AB \sin 0^\circ\,\hat{n} = 0 \quad \text{and} \quad \mathbf{A} \times \mathbf{B} = AB \sin 180^\circ\,\hat{n} = 0$$

Vector product of a vector with itself is equal to zero. i.e.,

$$\mathbf{A} \times \mathbf{A} = AA \sin 0^\circ\,\hat{n} = 0$$

Similarly, we can show that the vector product of a unit vector with itself is zero. i.e.,

$$\hat{i} \times \hat{i} = \hat{j} \times \hat{j} = \hat{k} \times \hat{k} = 0$$

3) Vector product of two perpendicular vectors has maximum value, and is equal to the product of their magnitudes only. Hence:

$$\mathbf{A} \times \mathbf{B} = AB \sin 90^0 \hat{n} = AB\hat{n}$$

Similarly, for mutually perpendicular unit vectors, we can show:

$$\hat{i} \times \hat{j} = \hat{k}, \qquad \hat{j} \times \hat{k} = \hat{i}, \qquad \hat{k} \times \hat{i} = \hat{j}$$

4) Vector product of two vectors in terms of their rectangular components can be given as:

$$\mathbf{A} \times \mathbf{B} = (A_x \hat{i} + A_y \hat{j} + A_z \hat{k}) \times (B_x \hat{i} + B_y \hat{j} + B_z \hat{k})$$

By using the determinant and solving, we get:

$$\mathbf{A} \times \mathbf{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} \text{ \_\_\_\_\_\_ (2.10)}$$

OR $$\mathbf{A} \times \mathbf{B} = (A_yB_z - A_zB_y)\hat{i} + (A_zB_x - A_xB_z)\hat{j} + (A_xB_y - A_yB_x)\hat{k} \text{ \_\_\_\_\_\_ (2.11)}$$

Physical Significance of Vector Product:

Vector product gives us the area of a plane. For example, if two vectors **A** and **B** represent adjacent sides of a plane, as shown in the figure 2.7, then magnitude of **A x B** gives us the magnitude of that area (area may be rectangle or parallelogram). The unit vector $\hat{n}$ gives the direction of that area, which is normal to the plane and can be found by right hand rule.

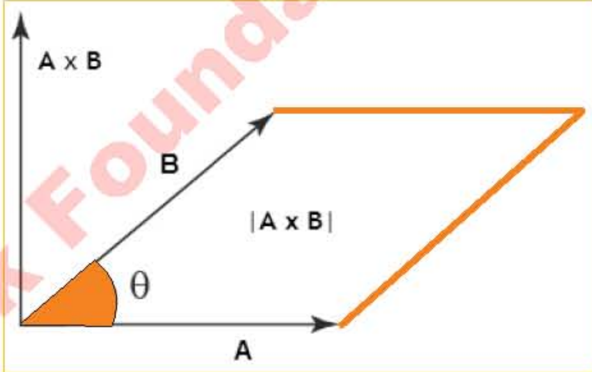


Figure 2.7: Magnitude of **A x B** gives area of parallelogram.

Example 2.3: Find the area of the parallelogram whose adjacent sides are given by the vectors: A = ($\hat{i}$ + 6 $\hat{j}$ + 2 $\hat{k}$) m and B = (7 $\hat{i}$ + $\hat{j}$ + 5 $\hat{k}$) m.

Given: **A** = ($\hat{i}$ + 6 $\hat{j}$ + 2 $\hat{k}$) m   **B** = (7 $\hat{i}$ + $\hat{j}$ + 5 $\hat{k}$) m

To Find: **A x B** = ?

Solution: As we know that vector product gives us the area of parallelogram. So,

$$\mathbf{A} \times \mathbf{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 6 & 2 \\ 7 & 1 & 5 \end{vmatrix}$$

$$\mathbf{A} \times \mathbf{B} = \hat{i} \begin{vmatrix} 6 & 2 \\ 1 & 5 \end{vmatrix} - \hat{j} \begin{vmatrix} 1 & 2 \\ 7 & 5 \end{vmatrix} + \hat{k} \begin{vmatrix} 1 & 6 \\ 7 & 1 \end{vmatrix}$$

$$\mathbf{A} \times \mathbf{B} = 28\hat{i} + 9\hat{j} - 41\hat{k} \text{ m}^2$$

Assignment 2.3

Two vectors A and B having magnitude 3.2 unit and 5.2 unit respectively, making an angle of 60° with each other. What is the magnitude of their cross products?

SUMMARY

- **Vectors** are those quantities, which require direction along with their magnitude for complete description.
- A vector with unit magnitude and arbitrary direction is called **unit vector**, it is required for the information of direction of any vector.
- A vector which specifies the location of a point with respect to origin is called **position vector**, and can be given as: $\mathbf{r} = a\,\hat{i} + b\,\hat{j}$ and its magnitude is given as: $r = \sqrt{a^2 + b^2}$.
- The components of a vector which are mutually perpendicular are called **rectangular components** of that vector.
- A vector can be found, if its rectangular components are given by relations: $A = \sqrt{(A_x)^2 + (A_y)^2}$ for magnitude and $\theta = \tan^{-1}\frac{A_y}{A_x}$ for direction.
- If a vector is given then its components can be found by the relation, $A_x = A\cos\theta$ for horizontal component and $A_y = A\sin\theta$ for vertical component.
- When the product of two vectors gives us scalar quantity, such product is called **scalar product**, mathematically given as: $\mathbf{A} \cdot \mathbf{B} = AB\cos\theta$.
- When the product of two vectors gives us vector quantity, such product is called **vector product**, mathematically given as: $\mathbf{A} \times \mathbf{B} = AB\sin\theta\,\hat{n}$.

EXERCISE

Multiple Choice Questions

Encircle the Correct option.

1) The number of perpendicular components of a force (in 2-D) are:

   A. 1 B. 2 C. 3 D. 4

2) $\hat{j} \times \hat{i} =$ ______

   A. 0 B. 1 C. $\hat{k}$ D. $-\hat{k}$

3) A force of 10 N is making an angle of 30° with the horizontal. Its x-components will be:

   A. 4 N B. 5 N C. 7 N D. 8.7 N

4) If two forces of magnitude 3 N and 4 N are acting at right angle to each other than their resultant force will be:

   A. 7 N B. 5 N C. 1 N D. Null vector

5) Angle between two vectors A and B can be easily determined by:

A. dot product B. cross product C. head to tail rule D. right hand rule

6) For which angle the equation $|\vec{A}.\vec{B}| = |\vec{A} \times \vec{B}|$ is correct?

A. 30° B. 45° C. 60° D. 90°

Short Questions

Give short answers of the following questions.

i) If the cross product of two vectors vanishes, what will you say about their orientation?
ii) Find the dot product of unit vectors with each other at (a) 0° and (b) 90°.
iii) Show that scalar product obeys commutative property.
iv) Solve by using the properties of dot and cross product: (a) $\hat{\imath} \cdot (\hat{\jmath} \times \hat{k})$ (b) $\hat{\jmath} \times (\hat{\jmath} \times \hat{k})$?
v) If both the dot product and the cross product of two vectors are zero. What would you conclude about the individual vectors?
vi) What are rectangular components of a vector? How they can be found?
vii) Give two examples for each of the scalar and vector product.
viii) Show that: $\mathbf{A} \cdot \mathbf{B} = A_x B_x + A_y B_y + A_z B_z$.
ix) What units are associated with the unit vectors $\hat{\imath}$, $\hat{\jmath}$, and $\hat{k}$?

Comprehensive Questions

Answer the following questions in detail.

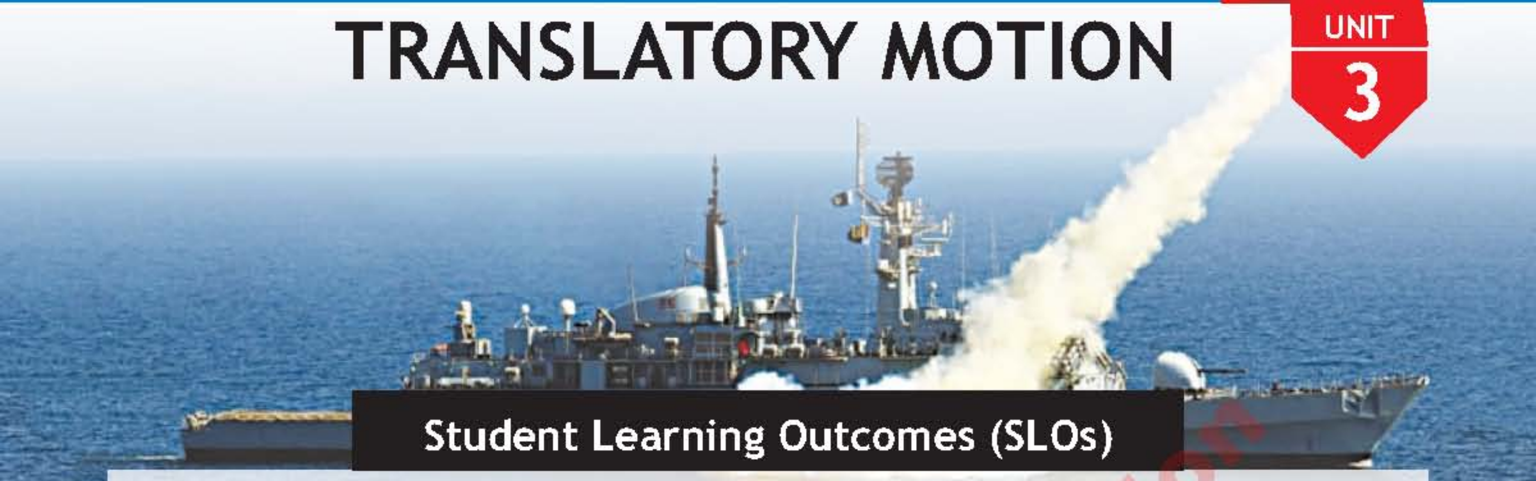
1) Explain the resolution of a vector into its rectangular components?
2) What is scalar product? Explain. Also write the properties of scalar product.
3) What is vector product? Explain. Also write the properties of vector product.
4) What geometric interpretation does the cross product have? Explain with the help of diagram.

Numerical Problems

1) If the magnitude of cross product between two vectors is $\sqrt{3}$ times the dot product, find angle between them. **(Ans: 60°)**
2) A force is acting on a body making an angle of 30° with the horizontal. The horizontal component of the force is 20 N. Find the force. **(Ans: 23.1 N)**
3) A vector having magnitude 5.5 N makes 10° with x-axis and vector $\vec{r}$ with magnitude 4.3 m makes 80° with x-axis. What is the magnitude of their dot and cross products? **(Ans: 8.1 Nm and 22.2 Nm)**
4) The magnitude of dot and cross product of two vectors are $6\sqrt{3}$ and 6 Respectively. Find the angle between the vectors. **(Ans: θ = 30°)**

TRANSLATORY MOTION

UNIT 3



Student Learning Outcomes (SLOs)

The students will:

- Derive the equations of motion [For uniform acceleration cases only. Derive from the definitions of velocity and acceleration as well as graphically].
- Solve problems using the equations of motion [For the cases of uniformly accelerated motion in a straight line, including the motion of bodies falling in a uniform gravitational field without air resistance. This also includes situations where the equations of motion need to be resolved into vertical and horizontal components for 2-D motion].
- Evaluate and analyse projectile motion in the absence of air resistance [This includes solving problems making use of the below facts:

  (i) Horizontal component (V) of velocity is constant.

  (ii) Acceleration is in the vertical direction and is the same as that of a vertically free falling object.

  (iii) The horizontal motion and vertical motion are independent of each other. Situations may require students to determine for projectiles:

  - How high does it go?

  - How far would it go along the level land?

  - Where would it be after a given time?

  - How long will it remain in flight?

  Situations may also require students to calculate for a projectile launched from ground height the

  - launch angle that results in the maximum range.

  - relation between the launch angles that result in the same range.
- Predict qualitatively how air resistance affects projectile motion [This includes analysis of both the horizontal component and vertical component of velocity and hence predicting qualitatively the range of the projectile.]
- Apply the principle of conservation of momentum solve simple problems.
- [Including elastic and inelastic interactions between objects in both one and two dimensions. Knowledge of the concept of coefficient of restitution is not required.
- Examples of applications include:

  - karate chops to break a pile of bricks

  - car crashes

  - ball & bat

  - the motion under thrust of a rocket in a straight line considering short thrusts during which the mass remains constant]
- Predict and analyse motion for elastic collisions [This include making use of the fact that for an elastic collision, total kinetic energy is conserved and the relative speed of approach is equal to the relative speed of separation]
- Justify why though the momentum of a closed system is always conserved, some change in kinetic energy may take place.

The cover photo of this chapter shows an anti-ship missile being fired by Pakistan Navy in the North Arabian Sea. The motion of a missile through the air can be described by its range, velocity, and acceleration. When it flies, a short patch of its path can be considered a straight line without any change in direction. But its motion is not straight for long, instead it is a 2-dimensional motion under the action of gravity, called projectile motion. Likely, all the objects in this universe are in motion. So, understanding of motion is also key to understand other concepts in physics. For example, an understanding of velocity is crucial to the study of momentum. There are three types of motion: translational motion, rotational motion and vibrational motion. In this chapter, we concern only with translational motion, such as motion on a straight line and projectile motion.

We begin with kinematics which is the branch of mechanics that deals with the study of motion without considering the causes of motion. This includes the terms such as displacement, velocity and acceleration in straight line. Using these terms, we study the motion of objects undergoing constant acceleration. We begin with derivation of the equations of motion and then we use them to solve the problems of uniformly accelerated motion in a straight line, including the motion of bodies falling in a uniform gravitational field without air resistance. It is also extended for objects moving along curved paths, such as projectile motion under the action of gravity only and its applications. Further deals with dynamics which is the branch of mechanics which deals with the study of forces that causes the objects and systems to move. This includes the term momentum and its law of conservation.

3.1 EQUATIONS OF UNIFORMLY ACCELERATED MOTION

There are three equations of motion which shows the relation between initial velocity, final velocity, acceleration, displacement and time. These equations describe and predict the motion of objects under constant acceleration. To simplify the derivation of these equations, following **assumptions** are made.

- Object is moving along the straight line.
- Acceleration is constant.
- Only magnitudes of vectors such as displacement, velocity and acceleration are considered.
- Direction of initial velocity and all the quantities which are in the direction of initial velocity are taken as positive.

Following are the three equations of motion:

$$v_f = v_i + a\,t$$

$$s = v_i\,t + \frac{1}{2}a\,t^2$$

$$2\,a\,s = (v_f)^2 - (v_i)^2$$

Let us derive these equations of motion.

3.1.1 First Equation of Motion

Consider a body moving with initial velocity v_i in a straight line with constant acceleration a. Its velocity becomes v_f after time t. The velocity-time graph for the motion of the body is shown in the figure 3.1. As we know that slope of the graph gives the acceleration of the body, so,

a = slope of line AB

OR $a = \frac{BC}{AC}$

OR $a = \frac{BD - CD}{OD}$ ______ (i)

from the graph it can be noted that:

BD represents the final velocity v_f

CD represents the initial velocity v_i

OD represents the time t. Hence we can write the equation (i) as:

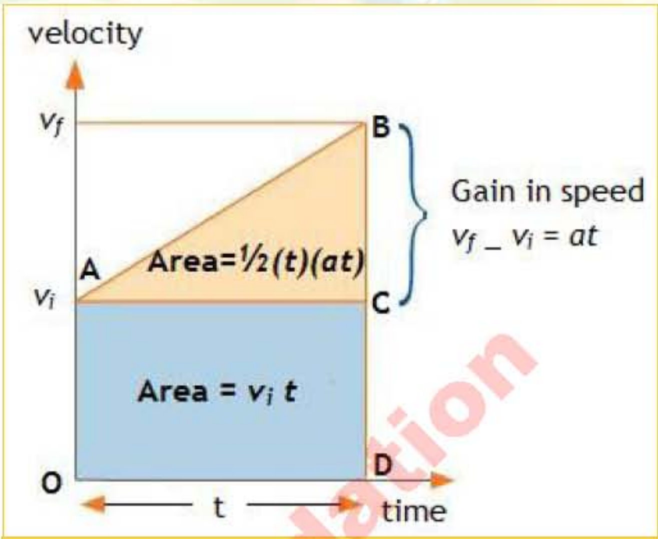


Figure 3.1: Velocity-time graph for a body moving with constant acceleration.

OR $a = \frac{v_f - v_i}{t}$

OR $at = v_f - v_i$

OR $v_f = v_i + at$ ______ (3.1)

Equation (3.1) is the first equation of motion, it represents the relation between initial velocity, final velocity, acceleration and time.

3.1.2 Second Equation of Motion

Consider a body moving with initial velocity v_i in a straight line with constant acceleration a. Its velocity becomes v_f after time t. The velocity-time graph for the motion of the body is shown in the figure 3.1.

As total distance S travelled by the body is equal to the total area under the graph. Hence:

Total distance S = Area of trapezium OABD

$$S = \frac{1}{2}\begin{pmatrix} \text{sum of} \\ \text{parallel sides} \end{pmatrix} \times \begin{pmatrix} \text{distance between} \\ \text{the parallel sides} \end{pmatrix}$$

As, from the graph we can see that OA and BD are parallel sides, and OD is the distance between these parallel sides, so using this we can write the above equation as:

$$S = \frac{1}{2}(OA + BD) \times OD$$

Putting OA = v_i, $BD = v_f$ and OD = t, we get:

$$S = \frac{1}{2}(v_i + v_f) \times t$$

Now using first equation of motion $v_f = v_i + at$, the equation may become:

$$S = \frac{1}{2}(v_i + v_i + a\,t) \times t$$

OR $S = \frac{1}{2}(2v_i + at) \times t$

OR $S = \frac{1}{2}(2\,v_i\,t + a\,t^2)$

Finally, we get:

$$S = v_i t + \frac{1}{2} a t^2 \qquad \text{\_\_\_\_\_\_ (3.2)}$$

Equation (3.2) is the second equation of motion, it represents the relation between initial velocity, acceleration, distance and time.

3.1.3 Third Equation of Motion

Consider a body moving with initial velocity v_i in a straight line with constant acceleration a. Its velocity becomes v_f after time t. The velocity-time graph for the motion of the body is shown in the figure 3.1.

As total distance S travelled by the body is equal to the total area under the graph. Hence:

Total distance S = Area of trapezium OABD

$$S = \frac{1}{2}\begin{pmatrix}\text{sum of}\\ \text{parallel sides}\end{pmatrix} \times \begin{pmatrix}\text{distance between}\\ \text{the parallel sides}\end{pmatrix}$$

As, from the graph we can see that OA and BD are parallel sides, and OD is the distance between these parallel sides, so using this we can write the above equation as:

$$S = \frac{1}{2}(OA + BD) \times OD$$

OR $2S = (OA + BD) \times OD$

Multiplying both sides by $\frac{BC}{OD}$, we get:

$$2S \times \frac{BC}{OD} = (OA + BD) \times OD \times \frac{BC}{OD}$$

As, $\frac{BC}{OD} = a$, so we get:

$$2S \times a = (OA + BD) \times BC$$

Putting $OA = v_i$, $BD = v_f$ and $BC = v_f - v_i$, we get:

$$2aS = (v_i + v_f) \times (v_f - v_i)$$

OR $2aS = v_f^2 - v_i^2$ ______ (3.3)

Equation (3.3) is the third equation of motion, it represents the relation between initial velocity, final velocity, acceleration and distance.

Example 3.1: A bus is travelling at 10 m/s accelerates uniformly at 2 m/s. Calculate the velocity of the bus after 5 s.

Given: initial velocity of the bus = v_i = 10 m/s acceleration = a = 2 m/s

Time = t = 5 s

To Find: Final velocity of the bus = v_f = ?

Solution: Using 1st equation of motion:

$v_f = v_i + a\,t$

Putting values and solving, we get:

$v_f = 10 + 2(5)$

$v_f = 20$ m/s

Example 3.2: A car is travelling initially with 4 m/s then it accelerates at 1 ms^{-2} for 10 s. Find the distance travelled by the car during this time.

Given: initial velocity of the car = v_i = 4 ms^{-1} acceleration = a = 1 ms^{-2}

time = t = 10 s

To Find: Distance travelled = S = ?

Solution: Using 2nd equation of motion:

$$S = v_i t + \frac{1}{2} a t^2$$

Putting values and solving, we get:

$$S = 4(10) + \frac{1}{2}(1)(10)^2$$

$$S = 40 + \frac{1}{2}(100) = 40 + 50 = 90 \text{ m}$$

Assignment 3.1

1) A train slows down from 80 km/h with a uniform retardation of 2m/s². How long will it take to attain a speed of 20 km/h?

2) A car travels with a velocity of 5 m/s. It then accelerates uniformly and travels a distance of 50 m. If the velocity reached is 15 m/s, find the acceleration and the time to travel this distance.

3.1.4 Free-Fall Motion

An example of uniformly accelerated motion is the motion of a free-falling body. In the absence of air resistance, all objects (lighter or heavier) falling freely near the surface of Earth with same acceleration independent of their masses. This acceleration is called acceleration due to gravity denoted by g. It has the value of 9.81 ms^{-2} and is directed towards centre of Earth.

By substituting $a = g$ and $s = h$ in above equations (3.1), (3.2) and (3.3) we get the equations of motion for free-fall as given below:

$$v_f = v_i + g\,t$$

$$h = v_i\,t + \frac{1}{2} g\,t^2$$

$$2\,g\,h = (v_f)^2 - (v_i)^2$$

Using these equations, we can solve problems for the the motion of bodies falling in a uniform gravitational field without air resistance.

Example 3.3: A kangaroo can jump over an object 2.50 m high. (a) Considering just its vertical motion, calculate its vertical speed when it leaves the ground. (b) How long a time is it in the air?

Given: height = h = 2.50 m Final velocity = v_f = 0 m/s (At highest point)

To Find: (a) Initial velocity = v_i = ? (b) Total time in the air = T = ?

Solution: (a) From 3rd equation of motion:

$$2\,g\,h = v_f^2 - v_i^2$$
$$2\,(-9.8)\,(2.5) = 0 - v_i^2$$
$$v_i^2 = 49\ m/s^2$$
$$v_i = 7\ m/s$$

(b) Now using 1st equation of motion:

$$v_f = v_i + g\,t$$

Where t is the time to reach the maximum height, g will be negative for moving up, so

$$0 = 7 - 9.8\,t$$
$$t = 0.71\ s$$

So, the total time in air = T = 2 (0.71 s) = 1.42 s

Assignment 3.2

A 1 kg ball is dropped from top of the leaning tower of Pisa. The ball reaches the ground in 3.34 s. Find the (a) height of the tower (b) velocity of the ball when it strikes the ground.

3.2 PROJECTILE MOTION

Till now we have studied the motion of bodies in straight line either horizontal or vertical. There are many situations in which a body moves along a curved path in a plane having both components i.e., vertical as well as horizontal. For example, when a player throws a football to another player, it moves in a curved path as shown in figure 3.2. Similarly, when a stone is thrown horizontally in air from the top of building, a long jumper leaves the ground at some angle, etc. are the examples of projectile motion.

Figure 3.2: 2-dimensional motion of a football.

Projectile motion is a two-dimensional motion of an object thrown in the air under the action of gravitational force only.

In our discussion of projectile motion, the effects of other forces such as air friction and rotation of Earth are neglected. The object that is thrown is called projectile and its path is called its trajectory. A football, cricket ball, a baseball or an arrow are the examples of projectile.

Experimentally it can be proved that the horizontal and vertical motions are completely independent of each other.

Consider the motion of two different coloured balls, blue and green as shown in figure 3.3. The blue ball is dropped vertically downward while green ball is thrown horizontally at the same instant from a cliff. Neglecting air resistance, the two balls hit the ground at the same time. Thus, the key to analyse two-dimensional projectile motion is to break the motion into two motions, one along the horizontal and one along vertically.

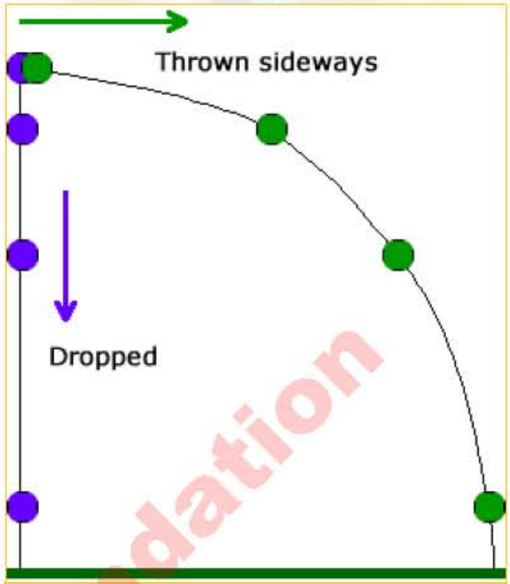


Figure 3.3: The two balls, one is thrown horizontally and other is thrown vertically down, hit the ground at the same time.

3.2.1 Projectile Motion for an Object Launched Horizontally

In many situations, when a projectile is thrown horizontally from certain height with some initial velocity, it travels forward as well as falls downward until it strikes ground. Hence, neglecting air resistance, the motion of body follows projectile motion.

Suppose the green ball which is thrown horizontally from a cliff of certain height with velocity v_i as shown in figure 3.3. There is no vertical component in the initial velocity.

Initial horizontal component of velocity = $v_{ix} = v_i$

Initial vertical component of velocity = $v_{iy} = 0$

There is no horizontal acceleration, while the vertical acceleration is the acceleration due to gravity, i.e.

$$a_x = 0, \qquad a_y = g$$

Acceleration due to gravity g is taken positive when the ball is coming downward and negative for the ball going upward.

The horizontal component of velocity remains constant, while the vertical component of velocity increases. So, at any instant t, its velocity components are:

Horizontal component of velocity is:

$$v_{fx} = v_{ix} = v_i cos\theta$$

Vertical component of velocity is:

$$v_{fy} = v_{iy} + a_y\, t$$

$$v_{fy} = 0 + g\, t$$

$$v_{fy} = g\, t$$

Hence magnitude of instantaneous velocity at any instant t is given by:

$$v_f = \sqrt{v_{fx}^2 + v_{fy}^2} \quad \text{\_\_\_\_\_\_ (3.4)}$$

The direction of velocity v_f is determined as:

$$\theta = tan^{-1}\left(\frac{v_{fy}}{v_{fx}}\right) \quad \text{\_\_\_\_\_\_ (3.5)}$$

At instant t, the **horizontal displacement** X covered by the ball is given by:

$$X = v_{fx}\, t \quad \text{\_\_\_\_\_\_ (3.6)}$$

And the **vertical displacement** the body moves downward from the height is given by:

$$Y = \frac{1}{2}\, g\, t^2 \quad \text{\_\_\_\_\_\_ (3.7)}$$

3.2.2 Projectile Motion for an Object Launched at Some Angle with Horizontal

There are many situations in which a projectile (object) is thrown with velocity v_i at an angle θ with the horizontal under the action of force of gravity as shown in figure 3.4. Here the air resistance is neglected.

There is no horizontal acceleration, while the **vertical acceleration is the acceleration due to gravity**, i.e.,

$$a_x = 0, \qquad a_y = -g$$

Resolve the initial velocity into its horizontal and vertical components,

$$v_{ix} = v_i cos\theta$$

$$v_{iy} = v_i sin\theta$$

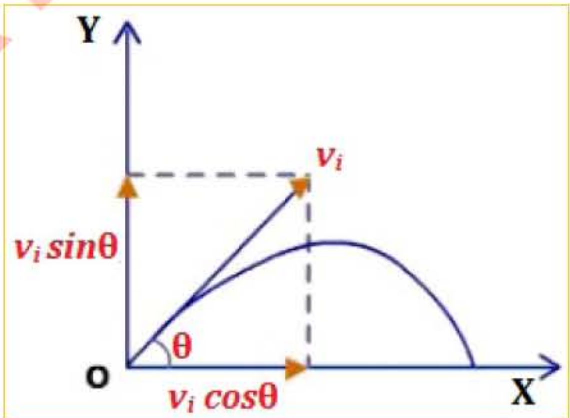


Figure 3.4: Rectangular components of initial velocity.

The horizontal component of velocity remains constant, while the vertical component of velocity changes. So, at any instant t, its velocity components are expressed as:

Horizontal component of velocity is:

$$v_{fx} = v_{ix} = v_i cos\theta$$

Vertical component of velocity is:

$$v_{fy} = v_{iy} + a_y\, t$$

$$v_{fy} = v_i sin\theta - g\, t$$

Hence, magnitude of instantaneous velocity at any instant t is given by:

$$v_f = \sqrt{v_{fx}^2 + v_{fy}^2}$$

The direction of velocity v_f is determined as:

$$\theta = tan^{-1}\left(\frac{v_{fy}}{v_{fx}}\right)$$

At instant t, the **horizontal displacement** x covered by the ball is given by:

$$x = v_{fx}\, t$$

$$x = v_i cos\theta \times t$$

And the **vertical displacement** the body moves in going up is given by:

$$y = v_{iy}\, t + \frac{1}{2}\, a_y\, t^2$$

$$y = (v_i sin\theta)\, t - \frac{1}{2}\, g\, t^2 \qquad \text{\_\_\_\_\_\_ (3.8)}$$

We will describe the maximum height, the range and time of flight of a projectile in the coming sections.

Height of the Projectile H:

The height of the projectile is the maximum vertical distance attained by it during the projectile motion. It is denoted by H as shown in figure 3.5. To determine H, we use the third equation of motion as;

$$2\, a_y H = v_{fy}^2 - v_{iy}^2 \text{ ------ (i)}$$

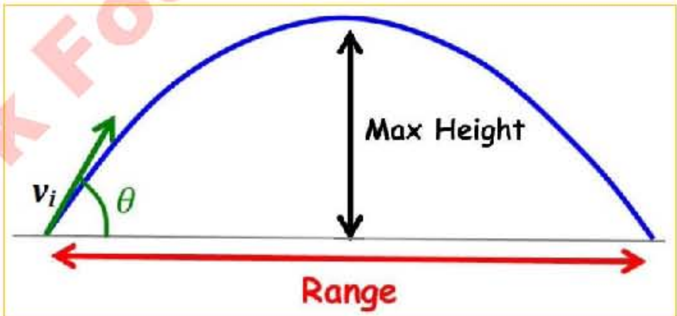


Figure 3.5: Height and range of a projectile.

It may be noted that at maximum height, $v_{fy} = 0$, since the body comes to rest at maximum height. Also, projectile moving upward, so $a_y = -g$, hence equation (i) becomes:

$$2(-g)H = 0 - v_{iy}^2$$

Also, as $v_{iy} = v_i\, sin\theta$ so, we get:

$$-2\, g\, H = 0 - v_i^2\, sin^2\theta$$

By arranging, we get:

$$H = \frac{v_i^2\, sin^2\theta}{2g} \qquad \text{\_\_\_\_\_\_ (3.9)}$$

Using this equation, we can find the height of projectile if magnitude and direction of initial velocity is known.

Time of flight T:

The time taken by the projectile from the point of projection to the point where it hits the ground at same level is called the time of flight. It is denoted by T. To determine T, we use the second equation of motion,

$$s = v_i t + \frac{1}{2} a t^2 \text{ ------ (i)}$$

For y-direction, equation (i) becomes:

$$H = v_{iy} t + \frac{1}{2} a_y t^2$$

As after time T the projectile comes back to ground (same height), so we take $H = 0$. Substituting $a_y = -g, v_{iy} = v_i \sin\theta$ and $t = T$ in above equation, we get:

$$0 = (v_i \sin\theta)\, T - \frac{1}{2}\, g\, T^2$$

By rearranging, we get:

$$T = \frac{2 v_i \sin\theta}{g} \quad \text{\_\_\_\_\_\_ (3.10)}$$

This equation can be used to find the time of flight, if magnitude and direction of initial velocity is known.

The time taken by the projectile to reach the highest point is called the time of summit. It is denoted by T' and is given by:

$$T' = \frac{T}{2} = \frac{v_i \sin\theta}{g} \quad \text{\_\_\_\_\_\_ (3.11)}$$

Range of the Projectile R:

The horizontal distance travelled by a projectile is called range. It is denoted by R. To determine R, we simply use the relation, $S = v\, t$. As the range is horizontal distance, so we can write:

$$X = v_x\, t$$

Substituting $X = R, v_x = v_i \cos\theta$ and $t = T = \frac{2 v_i \sin\theta}{g}$ in above equation, we get

$$R = v_i \cos\theta \times \frac{2\, v_i \sin\theta}{g}$$

Or $$R = \frac{v_i^2 \sin 2\theta}{g} \quad \text{---------- (3.12)}$$

This equation can be used to find range of a projectile, if magnitude and direction of initial velocity is known. Thus, only knowing two quantities; magnitude and direction of initial velocity, we can find height, range and time of flight for projectile.

Maximum Range R_{max}: The greater the initial speed, the greater is the range. For a given value of v_i the range of the projectile is maximum if $\sin 2\theta = 1$, which occurs when $2\theta = 90^o$ or $\theta = 45^o$. By substituting the value of $\theta = 45^o$ in equation (3.12), we get maximum range, i.e.

$$R_{max} = \frac{v_i^2}{g} \quad \text{______ (3.13)}$$

Same Range: If the speed of projectile v_i and g remains constant, then there are always two such angles for which the projectile has same range as shown in figure 3.6 (a). These angles are complementary angles of one another i.e., θ and 90° - θ. Hence a projectile has same range for pairs of angles (75°, 15°), (60°, 30°) and (70°, 20°) etc.

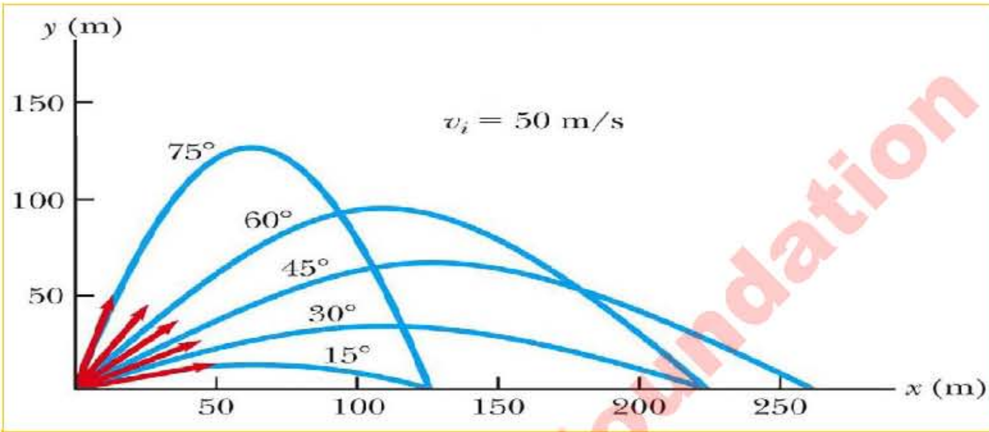


Figure 3.6 (a): Height verses range at different angles of projectile.

Effect of air resistance on projectile motion:

Generally, air resistance decreases the velocity of projectile. So as a result of air resistance both the horizontal component and vertical component of velocity decreases.

Air resistance affects the parabolic motion of a projectile by reducing its range and maximum height. Hence air resistance can significantly alter the trajectory of the motion as shown in figure 3.6 (b).

Air resistance also increases the time of flight of the projectile.

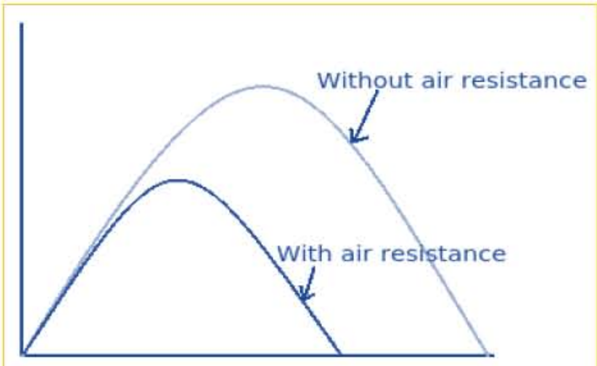


Figure 3.6 (b): Trajectory of a projectile with and without air resistance.

Example 3.4: If a projectile is launched horizontally with a speed of 12.0 m/s from the top of a 24.6 m high building. Determine the horizontal displacement of the projectile.

Given: v_{ix} = 12.0 m/s, y = 24.6 m

To Find: x = ?

Solution: First, we need to find time t, for this we use 2nd equation of motion in y-direction.

$$y = v_{iy}\,t + \frac{1}{2}a_y t^2$$

For a projectile launched horizontally:

v_{iy} = 0, a_y = g = 9.8 m/s^2, so we get:

$$24.6 = (0)t + \frac{1}{2}(9.8)\,t^2$$

$$t = 2.24\,s$$

Now using 2nd equation of motion in x-direction, we get:

$$x = v_{ix}\,t + \frac{1}{2}a_x\,t^2$$

For a projectile launched horizontally a_x = 0 m/s^2, so we get:

$$x = (12)\,(2.24) + \frac{1}{2}(0)t^2$$

$$x = 26.9\,m.$$

Example 3.5: A projectile is launched with an initial speed of 200.0 m/s at an angle of 30° above the horizontal.
(a) Determine the time of flight of the projectile.
(b) Determine the peak height of the projectile.
(c) Determine the horizontal displacement of the projectile.

Given: v_i= 200 m/s θ = 30°

To Find: (a) Time of flight = T = ? (b) Maximum height = H = ?
(c) Range of projectile = R = ?

Solution: (a) Time of flight = T = ?

$$T = \frac{2\,v_i \sin\theta}{g}$$

$$T = \frac{2\,(200)\,\sin 30}{9.8} = 20.41\text{ s}$$

(b) Maximum height = H = ?

$$H = \frac{v_i^2 \sin^2\theta}{2g}$$

$$H = \frac{(200)^2 \sin^2 30}{2(9.8)} = 510.2\text{ m}$$

(c) Range of projectile = R = ?

$$R = \frac{v_i^2 \sin 2\theta}{g}$$

$$R = \frac{(200)^2 \sin 2(30)}{9.8} = 3534.7\text{ m}$$

Assignment 3.3

A ball is thrown with a speed of 30 m/s in a direction 30° with the horizontal. Determine the:
(a) height to which it rises, (b) the time of flight and (c) the horizontal range.

3.3 LAW OF CONSERVATION OF MOMENTUM

If a system consists of particles each moving with different velocities, then the total momentum of an isolated system is the sum of the momenta of the individual particles. A system is said to be isolated if and only if the total external force, such as the gravitational force or friction, acting on the system is zero. There is no any ideally isolated system in the universe, but we consider an isolated system that does not interact with its environment. The law of conservation of momentum states as:

The total momentum of an isolated system of interacting particles is conserved.

If p_i and p_f are initial and final momentums of an isolated system then according to law of conservation of momentum:

$$p_f = p_i$$

OR $$p_f - p_i = 0$$

OR $$\Delta p = 0 \qquad (3.14)$$

The law of conservation of momentum is useful in collision problems. When a collision occurs in an isolated system due to internal forces, the momentum of each particle changes. Such forces do not contribute to the net force which remains zero. Hence, the total momentum of the system does not change with the passage of time. It is also applicable in explosions.

3.1.1 Applications of Conservation of Momentum

In an explosion, chemical energy (stored in the bonds of the atoms) is transformed into the kinetic energy of the fragments. Using the principles of conservation of momentum, many of the details of the explosion can be extracted and numerical values predicted for the fragments after the explosion.

All objects before an explosion are generally considered at rest. After and during the explosion, the objects fly away in different directions with different speeds. Momentum is always conserved, in this case, the initial momentum for everything is zero. Knowing this, the final momentum should be zero too.

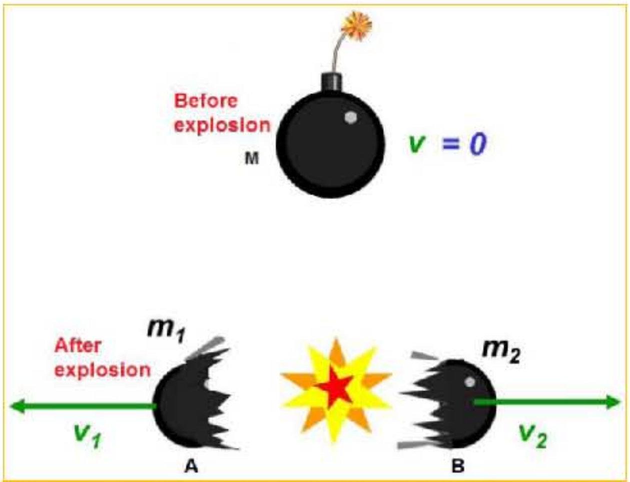


Figure 3.7: Explosion of a bomb into two fragments A and B.

Consider the explosion of a bomb into two fragments identified as A and B, as shown in figure 3.7. The system is the bomb and after explosion the two fragments are A and B. The initial momentum of the system before the explosion is zero. Momentum must be conserved, so sum of the final momentum of the two fragments must be zero. Therefore, the fragments must move in opposite directions with equal speed for their momentum to be conserved.

Explosion in a cannon also follow conservation of momentum. Consider cannon on a frictionless ground shooting a cannon ball as shown in figure 3.8.

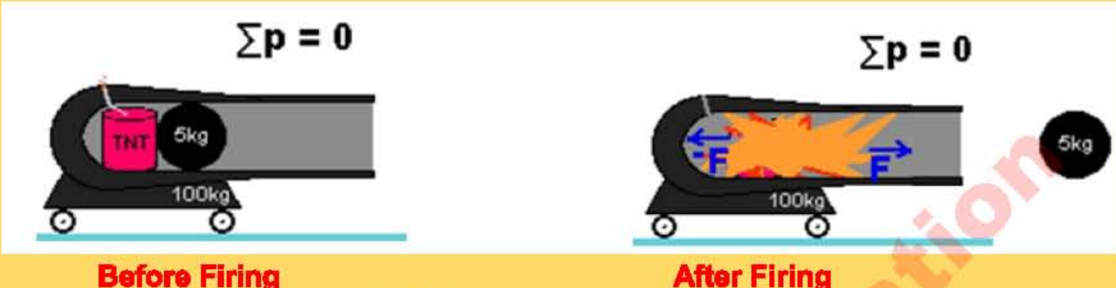


Figure 3.8: Momentum conservation in cannon.

The initial momentum is zero, because nothing is moving. After explosion inside the cannon, the cannon ball will be shot forward at very fast speed, while the cannon itself recoils in a much slower speed due to its heavy mass. Sum of the final momentum will also end up to zero, this makes the initial momentum and the final momentum the same.

In explosion objects move apart instead of coming nearer like collision.

Rockets and jet engines also work on the law of conservation of momentum. In these machines hot gases produced by burning of fuel rush out with large momentum. The machines gain an equal and opposite momentum. This enables them to move with very high velocity.

A **karate player** can break a pile of tiles with a single blow because he strikes the pile with his hand very fast. In doing so, the large momentum of his hand is reduced to zero in a very short time interval. This exerts a large force on the pile of tiles which is sufficient to break them apart.

Conservation of momentum is also applied on **ball and bat.** When a ball hits on the bat, there must be the same amount of momentum after the collision as there was before the collision. You have to add up the momentum of ball and bat. So the momentum of ball and bat before and after the collision must be equal.

Example 3.6: A 46 g tennis ball is launched from a 1.35 kg homemade cannon. If the cannon recoils with a speed of 2.1 m/s, determine the muzzle speed of the tennis ball.

Given: Mass of the ball = m_b = 46 g = 0.046 kg

Mass of the cannon = m_c = 1.35 kg

Recoil speed of the cannon = $v_c = -2.1$ m/s

To Find: Muzzle speed of the ball = v_b = ?

Solution: Initially, the ball is in the cannon and both objects are at rest. The total momentum of system is initially 0. i.e.,

$$p_i = 0$$

After the explosion, the total momentum of system must also be 0. Thus, the cannon's backward momentum must be equal to the ball's forward momentum.

$$p_f = 0$$
$$m_c v_c + m_b v_b = 0$$
$$(1.35\text{ kg})(-2.1\text{ m/s}) + (0.046\text{ kg})v_b = 0$$
$$-2.8 + (0.046)\,v_b = 0$$

Solving for v_b, we get:

$$v_b = 2.8/0.046 = 61.63\text{ m/s}$$

Assignment 3.4

A bullet of mass 20 g is fired from a gun with a muzzle velocity of 100 m/s. Find the recoil of the gun if its mass is 5 kg.

3.4 ELASTIC AND INELASTIC COLLISIONS

A collision occurs when two bodies come in physical contact with each other for a short interval of time and then separate. For example, the collision of ball with a bat, the collision of two cars etc. There are two types of collisions: Elastic collision and Inelastic collision. Momentum remains conserved in all types of collisions, but kinetic energy may change.

Inelastic Collision

Inelastic collision is such a collision in which the momentum is conserved but kinetic energy is not conserved.

For example, a meteorite falls on the Earth. In such collisions, the kinetic energy is transformed into other forms of energy, such as heat energy, sound energy.

Though the **momentum of a closed system is always conserved**, some change in kinetic energy may take place. While momentum of the system is conserved in an inelastic collision but kinetic energy is not. This is because some kinetic energy had been transferred to something else: thermal energy, sound energy, and material deformation etc.

Elastic Collision

Elastic collision is such a collision in which both the momentum and the kinetic energy of the system are conserved.

For example, the collision between atomic and subatomic particles is elastic. In such collision, the two objects collide and return to their original shapes with no loss of total kinetic energy, i.e. the kinetic energy does not change into other types of energy.

3.4.1 Elastic Collision in One Dimension

Consider an isolated system of two spherical bodies such as billiard balls of masses m_1 and m_2 initially moving towards right with velocities u_1 and u_2 along a straight line. The speed of m_1 is greater than that of m_2 and is approaching to it. So, the two bodies encounter head-on elastic collision. After collision they move with velocities v_1 and v_2 along the same straight line as shown in figure 3.9.

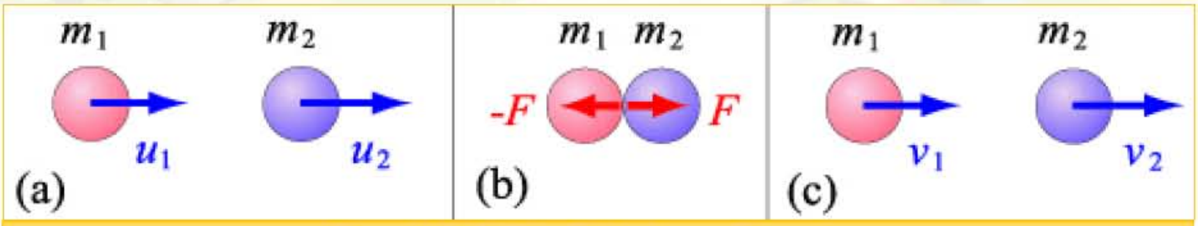


Figure 3.9: Elastic collision in one dimension.

We take the velocity as positive if it is moving towards right and negative if it is moving towards left. Since the collision is elastic, then in such situation we have two conservation equations. According to the law of conservation of momentum:

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2 \text{------ (i)}$$

After rewriting the above equation, we have:

$$m_1u_1 - m_1v_1 = m_2v_2 - m_2u_2$$

$$m_1(u_1 - v_1) = m_2(v_2 - u_2) \text{ ------ (ii)}$$

Similarly, according to the law of conservation of kinetic energy, we get:

$$\frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2 = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2$$

After cancelling the factor 1/2, rewrite the above equation:

$$m_1(u_1^2 - v_1^2) = m_2(v_2^2 - u_2^2)$$

$$m_1(u_1 - v_1)(u_1 + v_1) = m_2(v_2 - u_2)(v_2 + u_2) \text{------ (iii)}$$

Dividing equation (iii) by (ii), we get:

$$u_1 + v_1 = v_2 + u_2$$

Or $$u_1 - u_2 = v_2 - v_1$$

Or $$u_1 - u_2 = -(v_1 - v_2) \text{ \_\_\_\_\_\_ (3.15)}$$

This equation shows that relative speed of two bodies before collision is equal but opposite to relative speed after collision. Hence for two bodies colliding elastically, *relative speed of approach before collision is equal to relative speed of separation after collision.* From equation (3.15), we get:

$$v_2 = u_1 + v_1 - u_2 \quad \text{.......... (iv)}$$

By putting the value of v_2 from equation (iv) into the equation (i), we get:

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2(u_1 + v_1 - u_2)$$

Or $$m_1u_1 + m_2u_2 = m_1v_1 + m_2u_1 + m_2v_1 - m_2u_2$$

Or $$m_1v_1 + m_2v_1 = m_1u_1 + m_2u_2 - m_2u_1 + m_2u_2$$

Or $$(m_1 + m_2)v_1 = (m_1 - m_2)u_1 + 2m_2u_2$$

Or $$v_1 = \frac{(m_1 - m_2)}{(m_1 + m_2)}u_1 + \frac{2m_2}{(m_1 + m_2)}u_2 \text{ \_\_\_\_\_\_ (3.16)}$$

Similarly, from equation (iv)

$$v_1 = u_2 + v_2 - u_1 \quad \text{.......... (v)}$$

By putting the value of v_1 from equation (v) into the equation (i), we get:

$$m_1u_1 + m_2u_2 = m_1(u_2 + v_2 - u_1) + m_2v_2$$

Or $$m_1u_1 + m_2u_2 = m_1u_2 + m_1v_2 - m_1u_1 + m_2v_2$$

Or $$m_1v_2 + m_2v_2 = m_1u_1 + m_2u_2 - m_1u_2 + m_1u_1$$

Or $$(m_1 + m_2)v_2 = 2m_1u_1 - (m_1 - m_2)u_2$$

Or $$v_2 = \frac{2m_1}{(m_1+m_2)}u_1 - \frac{(m_1-m_2)}{(m_1+m_2)}u_2 \quad \text{———} \quad (3.17)$$

Equations (3.16) and (3.17) give the velocities of two bodies after collision.

Case 1: When bodies have the same mass i.e., $m_1 = m_2$, then from equations (3.16) and (3.17), we get:

$$v_1 = u_2 \qquad \text{and} \qquad v_2 = u_1$$

This shows that in one dimensional elastic collision, when two bodies of equal mass collide after the collision their velocities exchange.

Case 2: When bodies have the same mass i.e., $m_1 = m_2$, and second body (target) is at rest ($u_2 = 0$), then from equations (3.16) and (3.17), we get:

$$v_1 = 0 \qquad \text{and} \qquad v_2 = u_1$$

This shows that when the first body comes to rest the second body moves with the initial velocity of the first body.

Case 3: When a lighter body (m_1) collides with a massive body ($m_2 \gg m_1$) at rest ($u_2 = 0$), then under such condition m_1 can be neglected i.e., $m_1 \approx 0$, so from equations (3.16) and (3.17), we get:

$$v_1 = -u_1 \qquad \text{and} \qquad v_2 = 0$$

Hence the first body (which is lighter) rebounds with the same initial velocity as it has a negative sign. The second body (which is heavier) continues to remain at rest even after collision. For example, if a ball is thrown at a fixed wall, the ball will bounce back from the wall with the same velocity with which it was thrown but in opposite direction.

Case 4: When a massive body (m_1) collides with a lighter body ($m_1 \gg m_2$) at rest ($u_2 = 0$), then under such condition m_2 can be neglected i.e., $m_2 \approx 0$, so from equations (3.16) and (3.17), we get:

$$v_1 = u_1 \qquad \text{and} \qquad v_2 = 2\,u_1$$

Hence, the first body (which is heavier) continues to move with the same initial velocity. The second body (which is lighter) will move with twice the initial velocity of the first body.

Example 3.7: An object of mass 5 kg moving with a speed of 10 m/s collides with another object of mass 10 kg moving in the same direction with speed 5 m/s. Assume that the collision is a one-dimensional elastic collision. What will be the speed of both objects after the collision?

Given: m_1 = 5 kg $\quad u_1$ = 10 m/s $\quad m_2$ = 10 kg $\quad u_2$ = 5 m/s

To Find: v_1 = ? $\quad v_2$ = ?

Solution: Using the relation:

$$v_1 = \frac{(m_1-m_2)}{(m_1+m_2)}u_1 + \frac{2m_2}{(m_1+m_2)}u_2$$

Putting values: $$v_1 = \frac{(5-10)}{(5+10)}10 + \frac{2(10)}{(5+10)}5$$

$$= 50/15 = 3.3 \text{ m/s}$$

Similarly, $$v_2 = \frac{2m_1}{(m_1+m_2)}u_1 - \frac{(m_1-m_2)}{(m_1+m_2)}u_2$$

$$v_2 = \frac{2(5)}{(5+10)}10 - \frac{(5-10)}{(5+10)}5 = 75/15 = 5 \text{ m/s}$$

Hence, velocity of the lighter object will decrease to 3.3 m/s and heavier object will continue to move with the same velocity i.e., 5 m/s.

SUMMARY

- **Equations of motion** shows the relation between initial velocity, final velocity, acceleration, displacement and time.
- **Projectile motion** is a two-dimensional motion of an object thrown in the air under the action of gravitational force only. In projectile motion horizontal component of velocity remains constant, while the vertical component of velocity changes.
- The **height of the projectile** is the maximum vertical distance attained by it during the projectile motion.
- The time taken by the projectile to reach the maximum height and then return to the ground is called the **time of flight**.
- The horizontal distance travelled by a projectile is called **range.** Projectile has same range for complementary angles.
- The **law of conservation of momentum** states, the total momentum of an isolated system of interacting particles is conserved.
- **Elastic collision** is such a collision in which both the momentum and the kinetic energy of the system are conserved.
- **Inelastic collision** is such a collision in which the momentum is conserved but kinetic energy is not conserved.

EXERCISE

Multiple Choice Questions

Encircle the Correct option.

1) A projectile thrown upward moves in its parabolic path, the velocity and acceleration vectors for the projectile are perpendicular to each other at:

A. no where B. the highest point C. the launch point D. the landing point

2) A truck driving along a highway road has a large quantity of momentum. If it moves at the same speed but has twice as much mass, its momentum is _______.

A. zero B. quadrupled C. doubled D. unchanged

3) A 5 N force is applied to a 3 kg ball to change its velocity from 9 m/s to 3 m/s. This impulse causes the momentum change of the ball to be ____ $kg\ m/s$.

A. -2.5 B. -10 C. -18 D. -45

4) Which of the following statement is Not True for the horizontal motion of projectiles?

A. A projectile with a horizontal component of motion will have a constant horizontal velocity.

B. The horizontal velocity of a projectile is 0 m/s at the peak of its trajectory.

C. The horizontal velocity of a projectile is unaffected by the vertical velocity; these two components of motion are independent of each other.

D. The horizontal displacement of a projectile is dependent upon the time of flight and the initial horizontal velocity.

5) The vertical component of velocity of a projectile is smallest at:

A. The instant it is thrown. B. Halfway to the top.

C. The top. D. The landing point.

6) A 4 kg object has a momentum of 12 $kg\ m/s$. The object's speed is ___ m/s.

A. 3 B. 4 C. 12 D. 48

7) A bomb of mass 9 kg explodes into 2 pieces of mass $3kg$ and $6kg$. The velocity of mass 3 kg is 1.6 m/s, the kinetic energy of mass 6 kg is:

A. 3.84 J B. 9.6 J C. 1.92 J D. 2.92 J

8) which of the following statement is true about the projectile motion?

A. Projectile motion is the motion of an object projected vertically upward into the air and moving under the influence of gravity.

B. Projectile motion is the motion of an object projected into the air and moving independently of gravity.

C. Projectile motion is the motion of an object projected into the air and moving under the influence of gravity.

D. Projectile motion is the motion of an object projected horizontally into the air and moving independently of gravity.

14. What is the force experienced by a projectile after the initial force that launched it into the air in the absence of air resistance?

A. The gravitational force B. The nuclear force

C. The contact force D. The electromagnetic force

16. If a projectile is launched on level ground, what launch angle maximizes the range of the projectile?

A. 0° B. 30° C. 45° D. 90°

Short Questions

1) What are the conditions for using the equations of motion?
2) You through a small ball vertically up in the air. How are the velocity and acceleration of the ball oriented with respect to one another (a) when the ball is going upward (b) when the ball is coming downward?
3) For a projectile motion, is the velocity zero at any instant? Is the acceleration zero at any instant?

4) Construct motion diagrams showing the velocity and acceleration of a projectile at several points along its path, assuming (a) the projectile is launched horizontally and (b) the projectile is launched at an angle θ with the horizontal.

5) An aeroplane while flying horizontally drops a bomb when reaches exactly above the target, but missed it. Explain why?

6) What is the difference between explosion and collision? Give one example of each.

7) Why do a slow-moving loaded truck and a speeding rifle bullet each have a large momentum?

8) What is the difference between elastic and inelastic collision?

9) An object that has a small mass and an object that has a large mass have the same momentum. Which object has the largest kinetic energy?

10) Can objects in a system have momentum while the momentum of the system is zero? Explain your answer.

11) For any specific velocity of projection, prove that the maximum range is equal to four times of the corresponding height.

12) Is momentum conserved when a bat hits a ball.

Comprehensive Questions

1) Derive the equations of motion.
2) Define the law of conservation of momentum. Explain how conservation of momentum applies to a handball bouncing off a wall.
3) Explain elastic collision in one dimension and prove that for two bodies colliding elastically, relative speed of approach before collision is equal to relative speed of separation after collision.
4) Discuss the motion of a projectile under the following situations (a) an object launched at some angle with horizontal (b) an object launched horizontally.
5) What is projectile motion? Explain with the help of examples.
6) Derive mathematical equations for (a) Maximum height attained, (b) time of flight, (c) range of a projectile.

Numerical Problems

1) On dry concrete, a car can decelerate at a rate of 7.00 m/s^2, whereas on wet concrete it can decelerate at only 5.00 m/s^2. Compare the distances necessary to stop the car moving at 30.0 m/s (about 110 km/h) (a) on dry concrete and (b) on wet concrete.

(Ans: 64.3m / 90.0 m)

2) Find the angle of projection of a projectile for which the maximum height and corresponding range are equal? **(Ans: 76°)**

3) A jet plane comes in for a landing with a speed of 100 m/s, and its acceleration can have a maximum magnitude of 5.00 m/s^2 as it comes to rest.

(a) From the instant the plane touches the runway, what is the minimum time interval needed before it can come to rest?

(b) Can this plane land on a small tropical island airport where the runway is 0.8 km long? Explain your answer. **(Ans: 23 s, it cannot, it would need a long runway)**

4) A projectile is launched with an initial speed of 21.8 m/s at an angle of 35^oabove the horizontal.

(a) Determine the time of flight of the projectile.

(b) Determine the peak height of the projectile.

(c) Determine the horizontal displacement of the projectile.

(Ans: 2.55 s, 7.99 m, 45.6 m)

5) A projectile is launched horizontally from the top of a 45.2 m high cliff and lands a distance of 17.6 m from the base of the cliff. Determine the magnitude of the launch velocity.

(Ans: 5.79 m/s)

6) Two equal-mass carts roll towards each other on a level, low-friction track. One cart rolls rightward at 2 m/s and the other cart rolls leftward at 1 m/s. After the carts collide, they couple and roll together. Ignoring resistive forces, find their combined speed.

(Ans: 0.5 m/s)

7) A 0.5 kg ball traveling at a speed of 4 m/s to the right collides elastically with another ball of 3.5 kg which is initially at rest. Find velocities of both the balls after collision?

(Ans: −3.00 m/s, 1.00 m/s)

8) A 17.5 g bullet is fired at a muzzle velocity of 582 m/s from a gun with a mass of 8.0 kg and a barrel length of 75.0 cm.

(a) How long is the bullet travelled in the barrel?

(b) What is the force on the bullet while it is in the barrel?

(c) Find the impulse exerted on the bullet while it is in the barrel.

(d) Find the bullet's momentum as it leaves the barrel.

(Ans: 0.00258 s, 3950 N, 10.2 kg m/s, 10.2 kg m/s)

UNIT 4

ROTATIONAL AND CIRCULAR MOTION

Student Learning Outcomes (SLOs)

The students will:

- Express angles in radians.
- Define and calculate angular displacement, angular velocity and angular acceleration [This involves use of $S = r\,\theta$, $v = r\omega$, $\omega = 2\pi/T$, $a == r\omega^2$ and $a = v^2/r$ to solve problems].
- Use equations of angular motion to solve problems involving rotational motions.
- Analyse qualitatively motion in a curved path due to a perpendicular force.
- Define and calculate centripetal force [Use $F = mr\omega^2$, $F = mv^2/r$].
- Analyze situations involving circular motion in terms of centripetal force [e.g. situations in which centripetal acceleration is caused by a tension force, a frictional force, a gravitational force, or a normal force].
- Define and calculate moment of inertia of a body and angular momentum.
- State and apply the law of conservation of angular momentum. Illustrate the applications of conservation of angular momentum in real life. [Such as by flywheels to store rotational energy, by gyroscopes in navigation systems, by ice skaters to adjust their angular velocity].
- Justify how a centrifuge is used to separate materials using centripetal force.
- Derive and apply the relation between torque, moment of inertia and angular acceleration.
- Explain why the objects in orbiting satellites appear to be weightless.
- Describe how artificial gravity is created to counter weightlessness.

UNIT 4 ROTATIONAL AND CIRCULAR MOTION

Rotational motion is the turning or spinning motion of an object about an axis that passes through. For rotational motion of rigid objects (non-deformable and the particles forming it stay in fixed positions relative to one another, as the object is rotated) we consider an axis of rotation. Axis of rotation is a line about which rotation takes place. This line remains fixed during rotational motion, while the other points of the body move in circles about it.

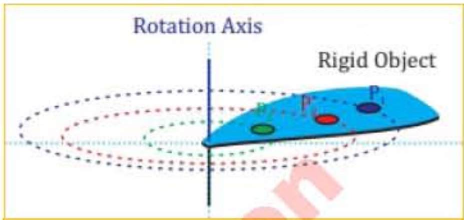


Figure 4.1: Axis of rotation.

Axis of rotation may be a pivot, hinges or any other support. Every point in a rotating rigid object moves in a circle (shown dashed in figure 4.1 for points P_1, P_2 and P_3) having center on the axis of rotation. A straight line drawn from the axis to any point in the object sweeps out the same angle in the same time interval.

4.1 ROTATIONAL KINEMATICS

Rotational kinematics deals with motion of objects along circular path without any reference to forces or torques.

4.1.1 Angular Position (θ)

The angle through which position vector of a moving object is displaced with respect to some chosen reference direction is called angular position 'θ'.

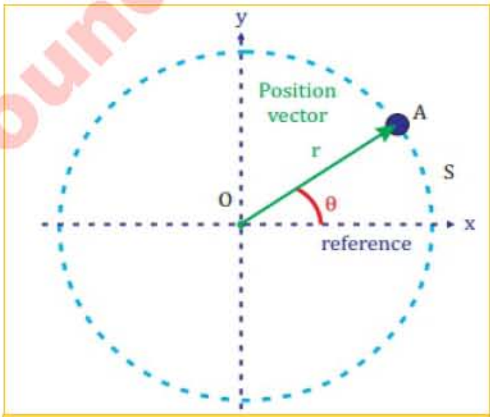


Figure 4.2: Angular Position.

Let an object 'A' is rotated through distance 'S' from a certain reference axis, along a circle of radius 'r' as shown in figure 4.2. The angular position of the rigid object is the angle 'θ' between this radial line (represented by position vector 'r') and the fixed reference line in space (often chosen as the + x axis). Mathematically,

$$\theta = \frac{s}{r} \quad \text{\_\_\_\_\_\_ (4.1)}$$

This resembles the way we identify the position of an object in translational motion as the distance x between the object and the reference position, which is the origin (x = 0).

4.1.2 Angular Displacement (Δθ)

The change in angular position with respect to chosen reference direction is termed as angular displacement.

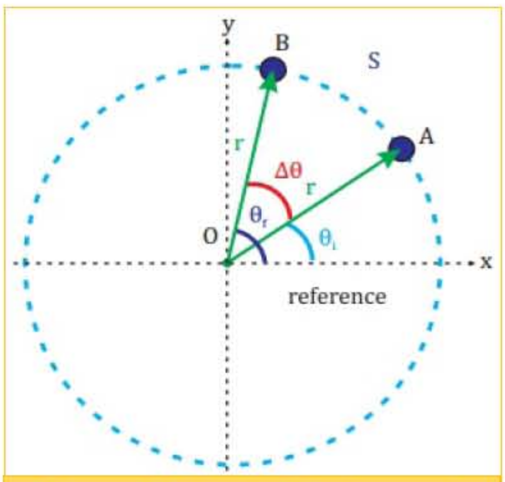
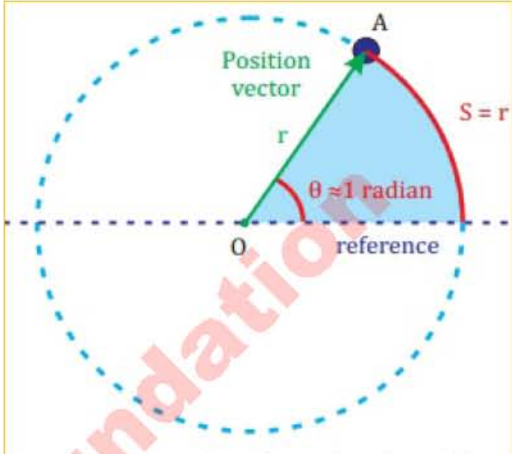


Figure 4.3: Angular Displacement.

As a particle on a rigid object travels from position A to position B in a time interval t as in Figure 4.3, the reference line fixed to the object sweeps out an angle, given by

$$\Delta \theta = \theta_{f\ i}\ \theta \_\_\_\_\_ (4.2)$$

Conventionally, positive angular displacements represent anti-clockwise motion and negative represents clockwise motion.



One **radian** (1 rad) is the angle subtended at the center of a circle by an arc with a length equal to the radius of the circle.

Figure 4.4: Radian Measurement.

Units of Angular Displacement: The SI units of angular displacement are radians. Other units are degrees and revolutions.

Relation between radian and degree:

In one complete rotation there are 360°.

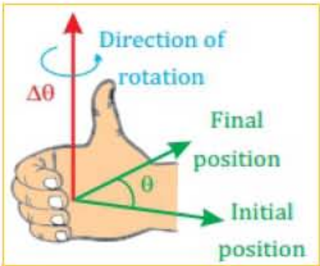
$$\text{Number of degrees in one revolution} = 360^0$$

To find the number of radians in one revolution, we put S as circumference of circle which is $2\pi r$, in equation 4.1, we get:

$$\text{Number of radians in 1 revolution} = 2\pi\ rad$$

As for one complete revolution the number of radians must be equal to the number of degrees, therefore:

$$2\pi\, rad = 360^0 \qquad \text{Or} \qquad 1 rad = \frac{360^0}{2\pi} \overset{0}{=} \frac{360}{2\times 3.14} = 57.3^0$$



Right Hand Rule

Direction of Angular Displacement: Angular displacement is a vector quantity; having both magnitude and direction. The right hand rule is used to specify the direction.

Holding the axis of rotation in right hand with fingers curling in the direction of rotation; the thumb gives the direction of angular displacement.

4.1.3. Angular Velocity (ω)

The time rate of angular displacement of a body is called angular velocity.

If '$\Delta\theta$' is the small angular displacement in time 'Δt' then angular velocity 'ω' is

$$\omega = \frac{\Delta \theta}{\Delta t} \_\_\_\_\_ (4.3)$$

Units of Angular velocity: The SI units of angular velocity are radian per second (rad/s).

Other units are deg/s or rev/s or rev /min (rpm). The direction of angular velocity is same as that of angular displacement.

Average Angular Velocity ω_{av}

The total angular displacement 'θ' of a body in time 't' is called average angular velocity.

$$\omega_{av} = \frac{\theta}{t} \quad \text{\_\_\_\_\_\_ (4.4)}$$

Instantaneous Average Velocity ω_{inst}

The limiting value of the ratio between small angular displacement '$\Delta\theta$' and small time interval 'Δt', such that the time approaches to zero is called instantaneous angular velocity.

$$\omega_{inst} = \lim_{\Delta t \to 0} \frac{\Delta\theta}{\Delta t} \quad \text{\_\_\_\_\_\_ (4.5)}$$

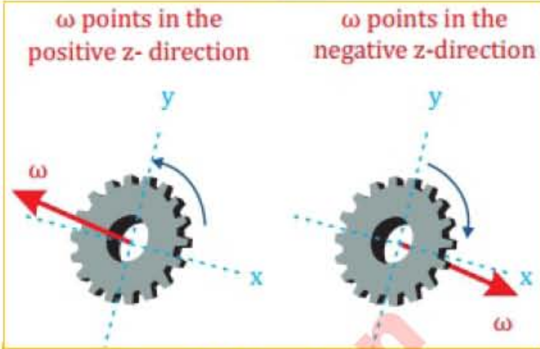


Figure 4.5: Angular velocity.

4.1.4. Angular Acceleration 'α'

The time rate of change of angular velocity is called angular acceleration.

If 'ω' is the angular velocity in time 't' then angular acceleration 'α' is

$$\alpha = \frac{\Delta\omega}{\Delta t} \quad \text{\_\_\_\_\_\_ (4.6)}$$

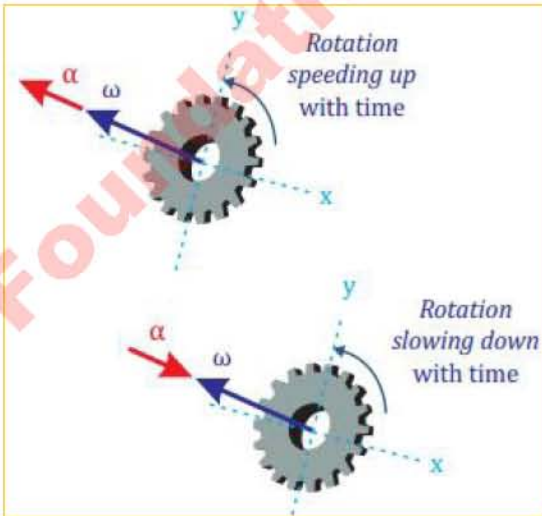


Figure 4.6: Angular acceleration.

Units of Angular Acceleration: The SI units of angular acceleration are rad/s^2. Other units are deg/s^2 or rev/sec^2.

The direction of angular acceleration is determined by right hand rule.

- It is taken as positive when angular velocity of a body increases. In such case angular velocity and angular acceleration have same direction.
- It is taken as negative when angular velocity of a body decreases. In such case angular velocity and angular acceleration are anti-parallel.

Average Angular Acceleration α_{av}

The total angular velocity 'ω' of a body in time 't' is called average angular acceleration.

$$\alpha_{avg} = \frac{\omega}{t} \quad \text{\_\_\_\_\_\_ (4.7)}$$

Instantaneous Average Acceleration α_{inst}

The limiting value of the ratio between small change in angular velocity 'ω' and small time interval 't', such that the time approaches to zero is called instantaneous angular acceleration.

$$\alpha_{inst} = \lim_{\Delta t \to 0} \frac{\Delta\omega}{\Delta t} \quad \text{\_\_\_\_\_\_ (4.8)}$$

4.1.5 Relationship between Linear and Angular Kinematic Quantities

Linear kinematic quantities like displacement, velocity and acceleration can be related with their rotational analogue.

A. Relation between Linear and Angular Displacement: Consider the Figure 4.7 in which a particle that moves in circle of radius 'r' with center at 'O'. Let the particle moves from point A to B, and there is another point C such that DAOC = 1 radian, therefore Arc AC must be equal to radius 'r'.

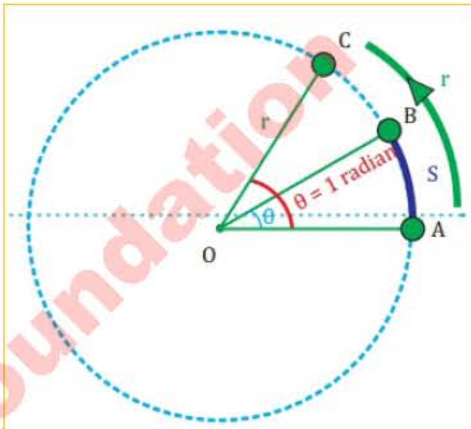


Figure 4.7: Linear and angular acceleration.

By using simple geometry, we can write:

$$\frac{Arc\,AB}{Arc\,AC} = \frac{Arc\,AOB}{Arc\,AOC} \_\_\_\_\_ (1)$$

Here, Arc AB is linear displacement 'S' and ∠AOC is the angular displacement 'θ'. As Arc AC = r and ∠AOC = 1 radian, so equation (1) becomes:

$$\frac{S}{r} = \frac{\theta}{1rad} \qquad \text{Or} \qquad \theta = \frac{S}{r}$$

For angular displacement θ in radians, we can write:

$$S = r\theta \_\_\_\_\_ (4.9)$$

B. Relation between Linear and Angular Velocity: Multiplying both sides of equation 1.9 by $\Delta/\Delta t$ and taking limit Δt approaches to zero. We get:

$$\lim_{\Delta t \to 0} \frac{\Delta}{\Delta t} S = \lim_{\Delta t \to 0} \frac{\Delta}{\Delta t} (r\theta) \_\_\_\_\_ (1)$$

Since there is no change in radius 'r' with respect to time, therefore:

$$\lim_{\Delta t \to 0} \frac{\Delta S}{\Delta t} = r \times \lim_{\Delta t \to 0} \frac{\Delta \theta}{\Delta t} \_\_\_\_\_ (2)$$

Now by definitions of linear and angular velocities:

$$v = \lim_{\Delta t \to 0} \frac{\Delta S}{\Delta t} \_\_\_\_\_ (3) \qquad \text{and} \qquad \omega = \lim_{\Delta t \to 0} \frac{\Delta \theta}{\Delta t} \_\_\_\_\_ (4)$$

Putting values from equation (3) and equation (4) in equation (2), we get:

$$v \neq \omega \_\_\_\_\_ (4.10)$$

The points A and B move closer together as Δt approaches to zero. And the direction of linear velocity is along the tangent to the circle. Therefore, this velocity is also called as tangential velocity.

C. Relation between Linear and Angular Acceleration: In angular motion the linear acceleration has two components tangential component and the radial component as shown in the figure 4.8.

In vector form: $a = a_T \; {}_R a$ ____ (1)

In magnitude $a = \sqrt{a_T^2 \; {}_R^2 a}$ ____ (2)

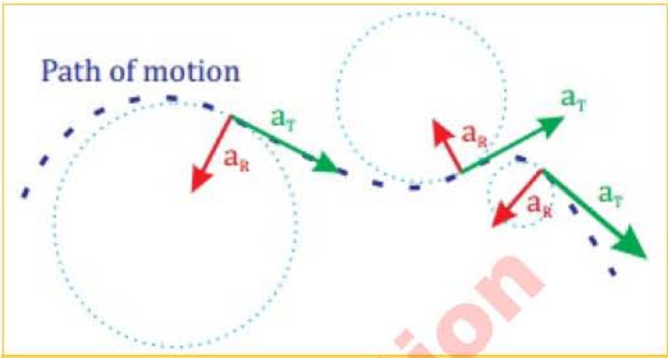


Figure 4.8: Acceleration components.

Tangential Component:

The component of angular acceleration which is parallel to linear instantaneous velocity is tangential component of acceleration. Thus tangential acceleration occurs due to change in magnitude of linear velocity. Multiplying both sides of equation 4.10 by $\Delta/\Delta t$ and taking limit Δt approaches to zero. We get:

$$\lim_{\Delta t \to 0} \frac{\Delta}{\Delta t} v = \lim_{\Delta t \to 0} \frac{\Delta}{\Delta t}(r\omega)$$

Since there is no change in radius 'r' with respect to time, therefore:

$$\lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} = r \times \lim_{\Delta t \to 0} \frac{\Delta \omega}{\Delta t} \text{ ____ (3)}$$

Now by definitions of linear and angular accelerations:

$$a = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} = a_T \text{ ____ (4)} \quad \text{and} \quad \alpha = \lim_{\Delta t \to 0} \frac{\Delta \omega}{\Delta t} \text{ ____ (5)}$$

putting values from equations (4) and (5) in equation (3), we get:

$$a_T = r\alpha \text{ ______ (4.11)}$$

This enables us to write all the kinematic equations in rotational form, as shown in table 4.1. Kinematics for rotational motion is similar to translational kinematics.

Table 4.1: Kinematic Equations for Rotational Motion

Equation for Linear Motion	Equation for Angular Motion
$S = v\,t$	$\theta = \omega\,t$
$v_f = v_i + at$	$\omega_f = \omega_i + at$
$2\,a\,S = v_f^2 - v_i^2$	$2\,\alpha\,\theta = \omega_f^2 - \omega_i^2$
$S = v_i\,t + ½\,at^2$	$\theta = \omega_i\,t + ½\,\alpha t^2$

Radial Component: The component of acceleration in angular motion which is along radius of the circular path is radial component of acceleration. This acceleration arises due to change in direction of linear instantaneous velocity. Thus for an object moving in a circular path with constant speed there is only the radial acceleration also called centripetal acceleration.

Example 4.1: In a workshop a bicycle tyre of radius 33.1 cm is rolled across the level floor with an initial velocity of 6.80 m/s. Assuming constant angular acceleration, the tyre comes to rest at a distance of 74.8 m. Determine (a) initial angular velocity of the tyre; (b) the total number of revolutions it made before coming to rest; (c) the angular acceleration of the tyre; and (d) the time it took before coming to rest.

Given: Initial velocity 'v_i' = 6.80 m/s Final angular velocity 'ω_f' = 0.00 rad/s
radius 'r' = 33.1 cm = 0.331 m distance 'S' = 74.8 m

To Find: (a) Angular velocity 'ω_i' = ? (b) Number of revolutions 'N' = ?
(c) Angular acceleration 'α' = ? (d) Time 't' = ?

Solution: (a) The relation between linear and angular velocity is $v = r\omega$ or $\omega = \frac{v}{r}$

Putting values: $\omega = \frac{6.80\ m/s}{0.331\ m}$ therefore, $\omega = 20.54\ rad/s$

(b) When the tyre completes one revolution, it moves a distance equal to the circumference of the tyre ($2\pi r$), as long as there is no slipping or sliding. The number of revolutions will be the total distance divided by distance covered during each revolution ($2\pi r$). $N = \frac{S}{2\pi r}$

Putting values: $N = \frac{74.8\,m}{2 \times 3.14 \times 0.331\,m}$ therefore, $N = 35.9\ rev$

(c) In one revolution there are 2π radians, the total angular displacement θ will be 35.9 × 2π radians = 225.6 radians (θ = 225.6 radians). To find angular acceleration we would use the equation independent of time (3rd equation) i.e.

$$2\alpha\theta = \omega_f^2 - \omega_i^2 \quad \text{Or} \quad \alpha = \frac{\omega_f^2 - \omega_i^2}{2\theta}$$

Putting values: $\alpha = \frac{(0\ rad/s)^2 - (20.54\ rad/s)^2}{2 \times 225.6\ rad}$ Or $\alpha = -0.94\ rad/s^2$

(d) To find 't', we can use any of the equation involving time, however the simpler equation $\omega_f = \omega_i + \alpha t$, by rearranging this equation for time, we get:

$$t = \frac{\omega_f - \omega_i}{\alpha}$$

putting values: $t = \frac{0\ rad/s - 20.54\ rad/s}{-0.94\ rad/s^2}$ therefore, $t = 21.9\ s$

So, the tyre will take about 22 seconds before coming to rest.

Assignment 4.1

The front wheel of a tractor travels 700 revolutions while the rear wheel 280 in a time interval of 40 seconds. Find their angular velocities.

4.2 CENTRIPETAL ACCELERATION AND CENTRIPETAL FORCE

Consider a particle which is moving in a circular path of radius r with constant speed, this means that direction of velocity is changing. This change in velocity of an object produces acceleration which is directed towards the center of the circle, such type of acceleration is called centripetal acceleration.

Consider the figure 4.9 (a) in which a particle follows a circular path. The particle is at point A at time t_i with velocity v_i. It is at point B at some later time t_f with velocity v_f. For uniform circular motion v_i and v_f differ only in direction their magnitudes are same, i.e. $|v_i| = |v_f| = |v|$

In figure 4.9 (b) velocity vectors have been redrawn tail to tail. The vector Δv joins the heads of two vectors, representing vector addition,

$$v_f = v_i + \Delta v$$

The angle 'Δθ' between the two position vector 'r_i' and 'r_f' is the same as the angle 'Δθ' between the two velocity vectors 'r_i' and 'r_f'. This is because the velocity vector is perpendicular to the position vector, thus the two angles must be same. This enable us to write the relation for the lengths of the sides of the two triangles.

$$\frac{\Delta v}{v} = \frac{\Delta r}{r}$$

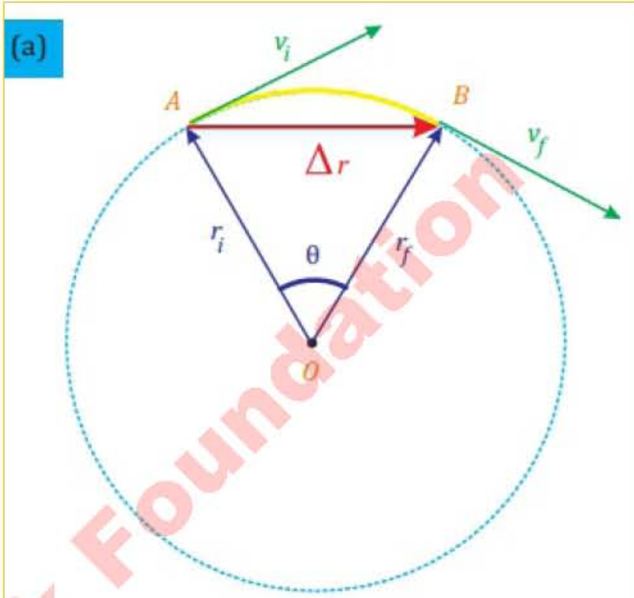
Where $|r_i| = |r_f| = |r|$ and $|v_i| = |v_f| = |v|$

Or $$\Delta v = v\frac{\Delta r}{r}$$

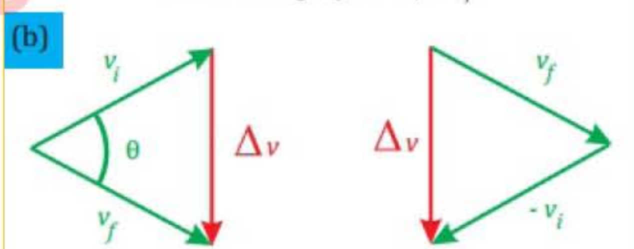
Dividing both sides by Δt, we get:

$$< a > = \frac{v}{r}\frac{\Delta r}{\Delta t}$$

Now imagine the points 'A' and 'B' in the figure to be extremely close together. As 'A' and 'B' approach each other 'Δt' approaches to zero. And acceleration at this stage will now be instantaneous acceleration



As the particle moves from point A to B its velocity vector changes from v_i to v_f.



Construction of determining the direction for change in velocity Δv. which is towards the center of the circle for small Δr.

Figure 4.9: Centripetal Acceleration.

$$a = \frac{v}{r}\lim_{\Delta t \to 0}\frac{\Delta r}{\Delta t}$$

Since, $v = \lim_{\Delta t \to 0}\frac{\Delta r}{\Delta t}$. Therefore, $$a = \frac{v}{r} \times v$$

This acceleration is referred to as centripetal acceleration a_c.

$$a = \frac{v^2}{r} \qquad \text{or} \qquad a_c = \left(\frac{v^2}{r}\right)\hat{r}$$

As centripetal acceleration a_c is directed towards the center of the circle, the radial vector r is directed outwards from the center of the circle thus a negative sign can be added with the equation.

$$a_c = -\left(\frac{v^2}{r}\right)\hat{r} \quad \text{\_\_\_\_\_\_ (4.12)}$$

as v = r ω, therefore, $a_c = \left(\frac{r^2\omega^2}{r}\right)\hat{r}$

Hence, $a_c = -r\omega^2\hat{r}$ ______ (4.13)

According to Newton's second law an object that is accelerating must have a net force acting on it. To open a door, force must be applied to produce tangential acceleration and thereby creating torque. Similarly, for an object moving in a circle, must also have a force applied to it to keep it moving in that circle. Thereby giving it necessary radial (centripetal) acceleration.

4.2.1 Centripetal Force

The net force that causes the particle to undergo centripetal acceleration is called centripetal force F_c.

When Newton's second law is applied to a particle moving in uniform circular motion, we can write:

$$\vec{F_c} = m\vec{a_c} \quad \text{\_\_\_\_\_\_ (1)}$$

Putting equation (4.12) or equation (4.13) in equation (1), we can write centripetal force F_c as:

$$\vec{F_c} = -\left(\frac{mv^2}{r}\right)\hat{r}$$

And $\vec{F_c} = -mr\omega^2\hat{r}$ ______ (4.14)

The direction of the centripetal force is always directed toward the center of the circle, and is continuously changing. Centripetal force is not a new force, but any net force that makes an object move towards the center of the circle can be termed as centripetal force. For example, to swing a ball in a circle on the end of a string, the tension in the string act as centripetal force. For a moon revolving around earth, or planets revolving around sun gravity act as centripetal force. In other cases, it can be a normal force, or even an electric force (as in CD players and computer hard disks).

Bending of Roads: When a car travels without skidding around an un-banked curve, the static frictional force between the tyres and the road provides the centripetal force. The reliance on friction can be eliminated completely for a given speed, if the roads are banked at an angle relative to the horizontal while making a turn (Figure 4.10). Because the roadbed makes an angle with respect to the horizontal, the normal force has a component 'F_N sinθ' that points toward the center 'C' of the circle and provides the centripetal force.

In the Figure 4.10 (a) part shows a car going around a friction-free banked curve. The radius of the curve is 'r', where 'r' is measured parallel to the horizontal. Part (b) of the figure shows the normal force 'F_N' that the road applies on the car, the normal force is perpendicular to the road. Because the roadbed makes an angle 'θ' with respect to the horizontal, the normal force has a component 'F_N sin θ' that points toward the center C of the circle and provides the centripetal force.

$$F_c = F_N = \sin\theta = \frac{mv^2}{r} \quad \_\_\_\_\_\_ (1)$$

Since the car does not accelerate along the component of normal force 'F_N cos θ', this component only balances the weight 'mg' of the car. Therefore, 'F_N cos θ = mg'. Dividing equation (1) by this equation, we get: $\frac{F_N \sin\theta}{F_N \cos\theta} = \frac{mv^2 / r}{mg}$

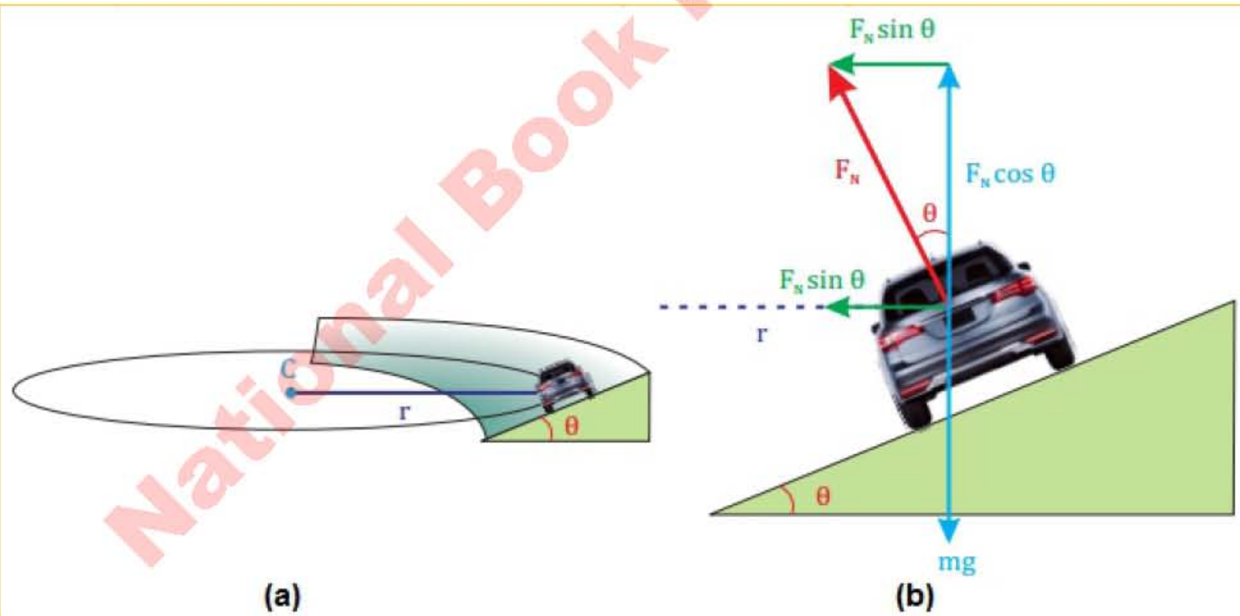


Figure 4.10

or $$\tan\theta = \frac{v^2}{rg} \quad \_\_\_\_\_\_ (4.15)$$

This Equation indicates that, for a given speed v, the centripetal force needed for a turn of radius 'r' at an angle 'θ' is independent of the mass of the vehicle. Greater speeds and smaller radii require more steeply banked curves—that is, larger values of 'θ'. At a speed that is too small for a given ‘θ’, a car would slide down a frictionless banked curve; at a speed that is too large, a car would slide off the top.

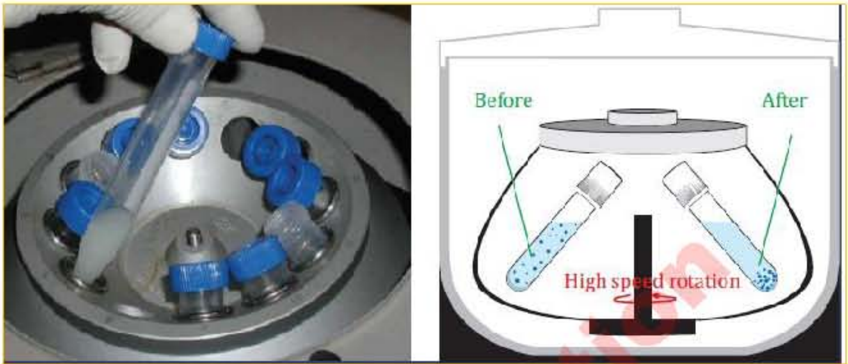


Figure 4.11

Centrifuge: A centrifuge is a device that separates substances suspended in a liquid mixture by spinning a sample of liquid mixture very quickly around an axle. Any small denser particles found in the liquid travel in a straight line inside the test tube, obeying Newton’s first law. The liquid in the test tube applies a centripetal force on these particles to keep them moving in a circle. After running the centrifuge at high speed for a period of time, the particles become clumped together at the bottom of the test tube, which can be collected and the sample is analyzed as shown in figure 4.11.

The same centrifugation principle can be used in these commonly used devices.

Cream Separator is a centrifugal device that separates milk into cream and skimmed milk.

Washing Machine Dryer consists of a long cylinder with small holes on its walls. Wet clothes are placed in this cylinder, and then rotated rapidly to dry it.

Example 4.2: The centripetal force on a car of mass 856 kg moving along a curve is 7250 N. If its speed is 12.0 m/s, what is the radius of the curve?

Given: mass ‘m’ = 856 kg Centripetal force ‘F_c’ = – 7250 N. Speed ‘v’ = 12.0 m/s,

Solution: Radius ‘r’ = ?

Solution: The centripetal force is given as:

$$F_c = -\frac{mv^2}{r} \quad \text{Or} \quad r = -\frac{mv^2}{F_c}$$

Putting values: $$r = -\frac{856\,kg \times (12\,m/s)^2}{-7250\,N} \quad \text{or} \quad r = 17\,m$$

Assignment 4.2

A car, that has centre of gravity 0.4 m above the earth’s surface, passing through a curve whose speed limit is 15 m/s. Find radius of the curve.

4.3 MOMENT OF INERTIA

The property of a body by which it maintains its state of rest or uniform rotational motion about a fixed axis is called moment of inertia (or rotational inertia).

The moment of inertia (or rotational inertia) is the rotational equivalent of mass. Objects with larger mass have a larger inertia, meaning that they are harder to accelerate. Similarly, an object with a larger moment of inertia is harder to angularly accelerate. The moment of inertia is given as:

$$I = mr^2 \quad \text{(4.16)}$$

If the body is rigid we divide the whole body into large number of small portions having masses $m_1, m_2, m_3, \ldots, m_n$ having radii $r_1, r_2, r_3, \ldots, r_n$ from its axis of rotation as shown in figure 4.12, and moment of inertia is given as:

$$I = \sum_{i=1}^{i=n} m_i r_i^2 \quad \text{(4.17)}$$

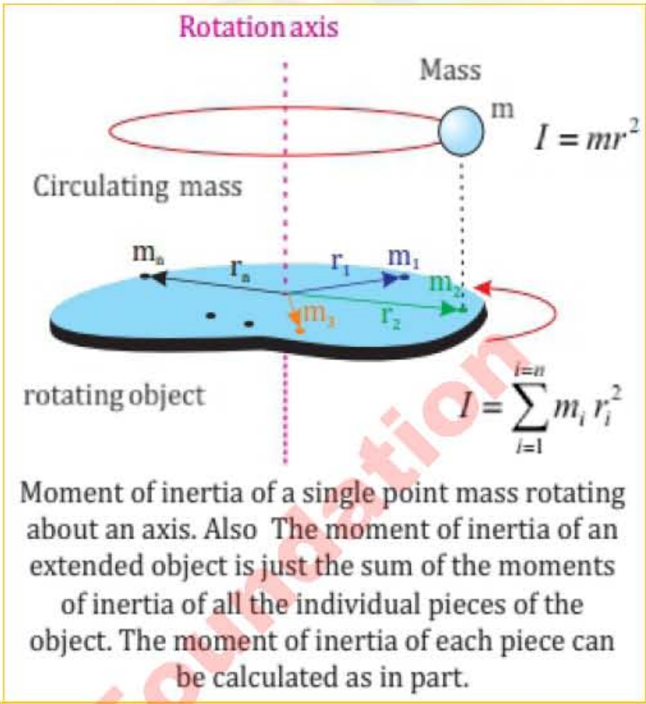


Figure 4.12: Moment of inertia.

Figure 4.13 shows the calculated moments of inertia for various objects of uniform composition, each with mass 'M'.

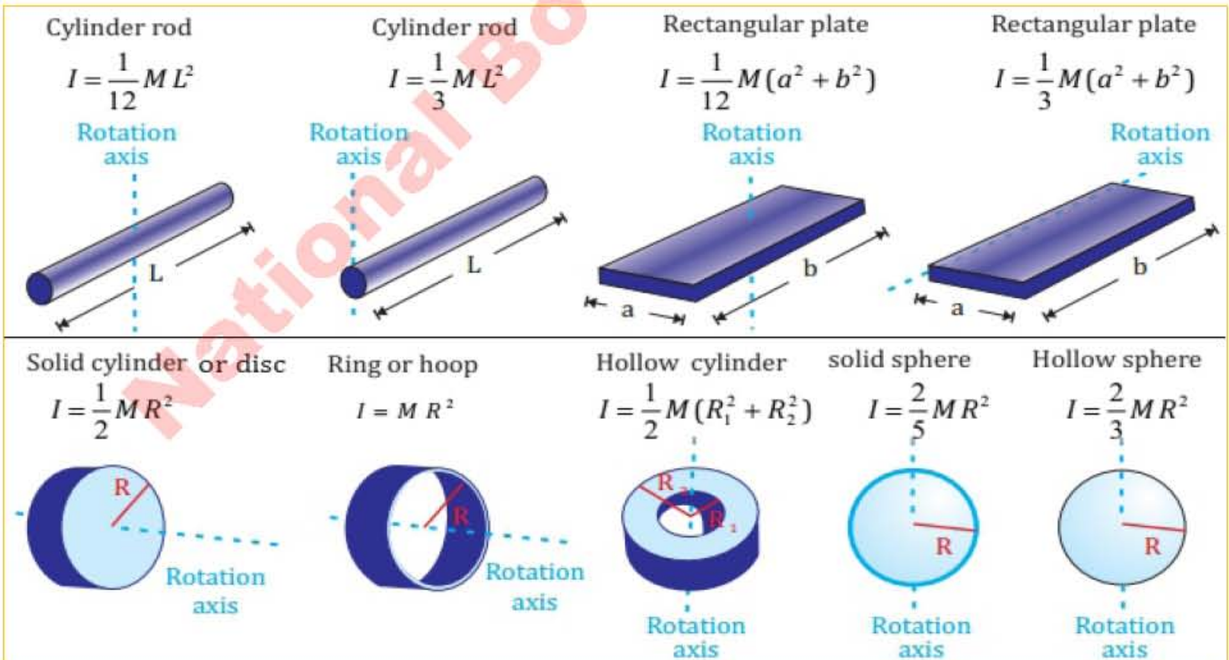


Figure 4.13: Moment of inertia for uniform objects.

4.4 ANGULAR MOMENTUM

The angular momentum 'L' of an object is defined as:

The cross product of position vector 'r' with respect to axis of rotation and linear momentum 'P' of an object. $\vec{L} = \vec{r} \times \vec{p}$

The SI unit of angular momentum is kgm^2s^{-1}, and dimensions are $[ML^2T^{-1}]$.

4.4.1. For a point mass

Consider a mass 'm' rotating at distance 'r' from the axis of rotation as shown in figure 4.14. By definition of Angular momentum:

$$\vec{L} = \vec{r} \times \vec{p} \quad \text{or} \quad \vec{L} = r\,p\sin\theta\,\hat{n}$$

Since θ = 90° and sin90°=1, therefore, magnitude of angular momentum is given by

$$L = r\,p \_\_\_\_\_\_ (1)$$

From the definition of linear momentum

$$p = mv \_\_\_\_\_\_ (2)$$

The relation between linear and angular velocity is

$$v \neq \omega \_\_\_\_\_\_ (3)$$

Putting equation (3) in equation (2), we get:

$$p = mr\omega \_\_\_\_\_\_ (4)$$

Now putting equation (4) in equation (1), we get:

$$p = r(mr\omega)$$

Hence $L = mr^2\omega = I\omega$ ______ (4.18)

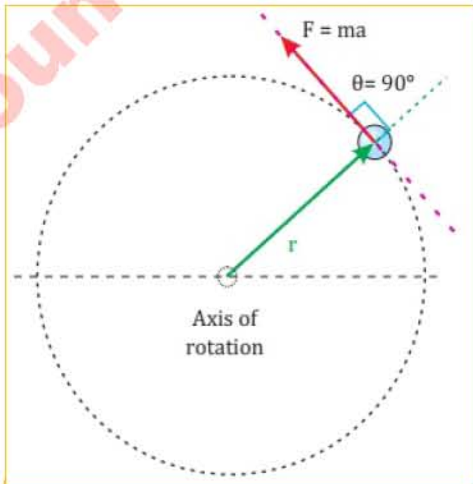


Figure 4.14: Mass 'm' rotating at distance 'r' from axis of rotation.

From equation 4.18, the angular momentum of an object can also be defined as the product of moment of inertia and its angular velocity, just like linear momentum is defined as product of mass and velocity.

4.4.2. For a Rigid Body

Consider a rigid body and divide it into large number of small masses 'm_1, m_2, m_3, ..., m_n' having distances 'r_1, r_2, r_3, ... rn' from the axis of rotation as shown in the figure 4.15. The net angular momentum will be sum of all the individual angular momentums

$L_{net} = L_1 + L_2 + L_3 + L_4 + ---- + L_n$ ___ (1)

The angular momentum about point 1 will be:

$$L_1 = m_1 r_1^2 \omega_1 \text{ \_\_\_\_\_ } (2)$$

Similarly, the angular momentum about point 2 will be:

$$L_2 = m_2 r_2^2 \omega_2 \text{ \_\_\_\_\_ } (3)$$

And $L_3 = m_3 r_3^2 \omega_3$ _____ (4)

Similarly, $L_n = m_n r_n^2 \omega_n$ _____ (5)

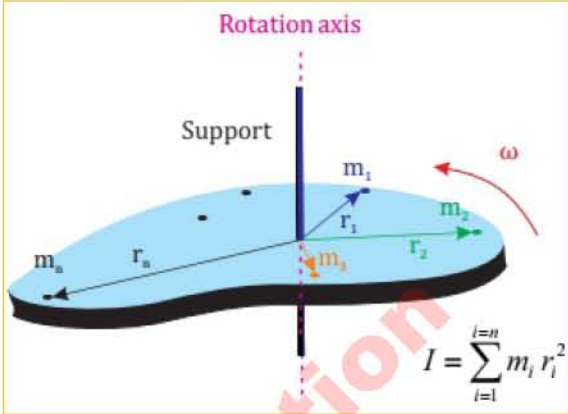


Figure 4.15: Rigid body rotating about axis of rotation.

Putting equations (2), (3), (4) and (5) in equation (1), we get:

$$L_{net} = m_1 r_1^2 \omega_1 + m_2 r_2^2 \omega_2 + m_3 r_3^2 \omega_3 + ----- + m_n r_n^2 \omega_n \text{ \_\_\_\_\_ } (6)$$

Since for same rigid body, all points on the body rotate with the same angular velocity 'ω', so

$$\omega_1 = \omega_2 = \omega_3 = ---- = \omega_n = \omega$$

Therefore, equation (6) can be written as:

$$L_{net} = m_1 r_1^2 \omega + m_2 r_2^2 \omega + m_3 r_3^2 \omega + ----- + m_n r_n^2 \omega$$

Or $$L_{net} = \left(m_1 r_1^2 + m_2 r_2^2 + m_3 r_3^2 + ----- + m_n r_n^2\right)\omega$$

The term in parenthesis in above equation is moment of inertia of a rigid body, so,

$$L_{net} = \left(\sum_{i=1}^{i=n} m_i r_i^2\right)\omega = I\omega \text{ \_\_\_\_\_ } (4.19)$$

4.4.3. Relation between Torque and Angular Momentum

The angular momentum L of an object is defined as:

The cross product of position vector r with respect to axis of rotation and linear momentum P of an object.

$$\vec{L} = \vec{r} \times \vec{p}$$

Multiplying both sides by $\frac{\Delta}{\Delta t}$, we get:

$$\frac{\Delta}{\Delta t}\vec{L} = \frac{\Delta}{\Delta t}(\vec{r} \times \vec{p}) \quad \text{Or} \quad \frac{\Delta \vec{L}}{\Delta t} = \vec{r} \times \frac{\Delta \vec{p}}{\Delta t} \text{ \_\_\_\_\_ } (1)$$

By Newton's second law of motion in terms of momentum: $F = \frac{\Delta p}{\Delta t}$ ______ (2)

Putting equation (2) in equation (1), we get: $\frac{\Delta L}{\Delta t} = r \times F$ ______ (3)

By the definition of torque $\tau = r \times F$ ______ (4)

Therefore, $\frac{\Delta L}{\Delta t} = \tau$ ______ (4.20)

4.4.4. Conservation of Angular Momentum

In the absence of any external torque, the angular momentum of the system remains constant.

I.e., $\frac{\Delta \vec{L}}{\Delta t} = 0$ ______ (1)

Therefore, $\Delta \vec{L} = 0$ ______ (2)

Or $\vec{L}_i - \vec{L}_f = 0$ ______ (3)

Hence, $\vec{L}_f = \vec{L}_i$

Or $I_f \vec{\omega}_f = I_i \vec{\omega}_i$ ______ (4.21)

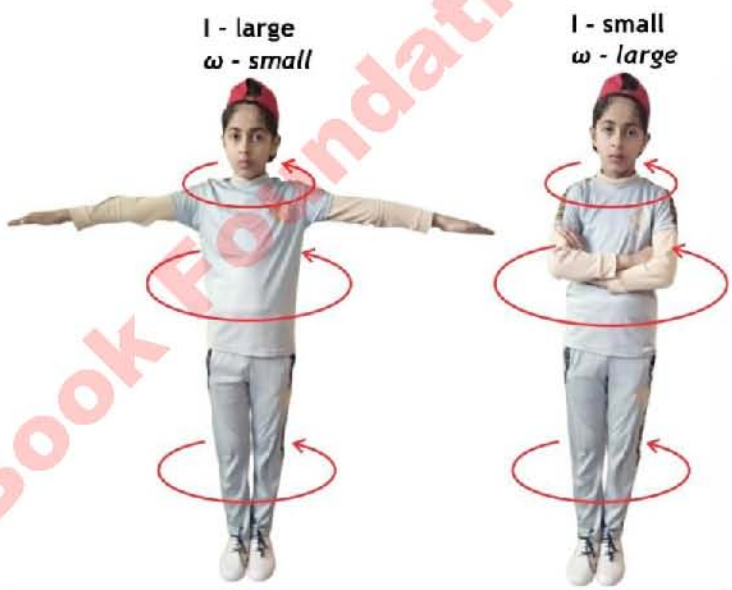


Figure 4.16: spinning ice skater.

Equation (4.21) implies that

The final angular momentum should be equal to initial angular momentum.

A spinning ice skater is an interesting example of conservation of angular momentum. When the skater's arms are extended, the rotational inertia, I, is relatively large and the angular velocity, is relatively small as shown in figure 4.16. Often at the end of the spin, the skater pulls her arms tight to the body resulting in a much faster spin (larger angular velocity) because of a much smaller rotational inertia, I. When a rotating body contracts, its angular velocity increases; and when a rotating body expands, its angular velocity decreases. This phenomenon is the result of the conservation of angular momentum. As

$$\vec{L}_f = \vec{L}_i \quad \text{or} \quad I_f \vec{\omega}_f = I_i \vec{\omega}_i$$

And moment of inertia is given by:

$I = mr^2$

Therefore, $m_f r_f^2 \vec{\omega}_f = m_i r_i^2 \vec{\omega}_i$

As $r_f < r_i$ and $m_f = m_i$ therefore her rotational speed will increase to compensate for the decrease in rotational inertia.

Similarly, gymnasts and divers generate their spins (torque) from a solid base or a diving board after which the angular momentum remains unchanged as shown in figure 4.17. The usual somersaults and twists result from making variations in their rotational inertia.

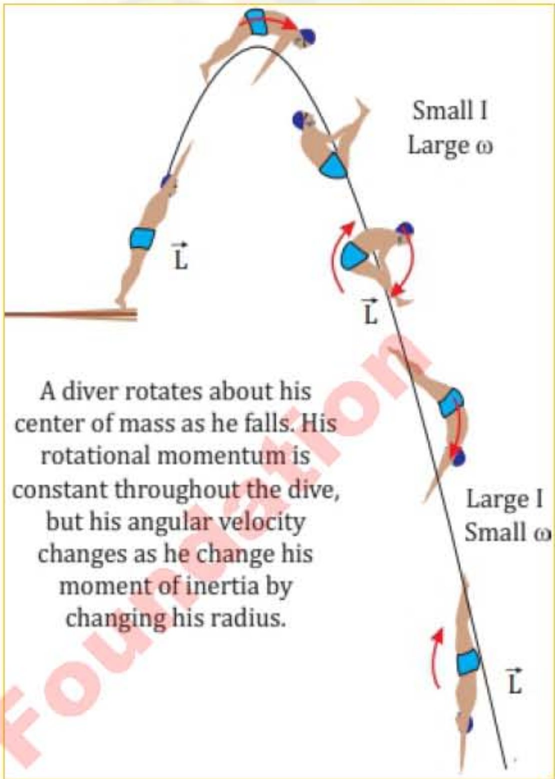


Figure 4.17: Board divers.

A **gyroscope** is a device that utilizes the principle of angular momentum to maintain its orientation relative to the Earth's axis or resist changes in its orientation. Gyroscope usually consist of a wheel mounted on an excel which can rotate freely secured in a metal frame as shown in figure 4.18 (a). When the wheel is made to spin the gyroscope can be mounted on flat surface, however as the wheel stop spinning the gyroscope falls. If the gyroscope is tilted it also keep levitated without falling, but will start precession about gravitational force axis. This is the gravity-defying part of a gyroscope, as shown in the figure 4.18 (b).

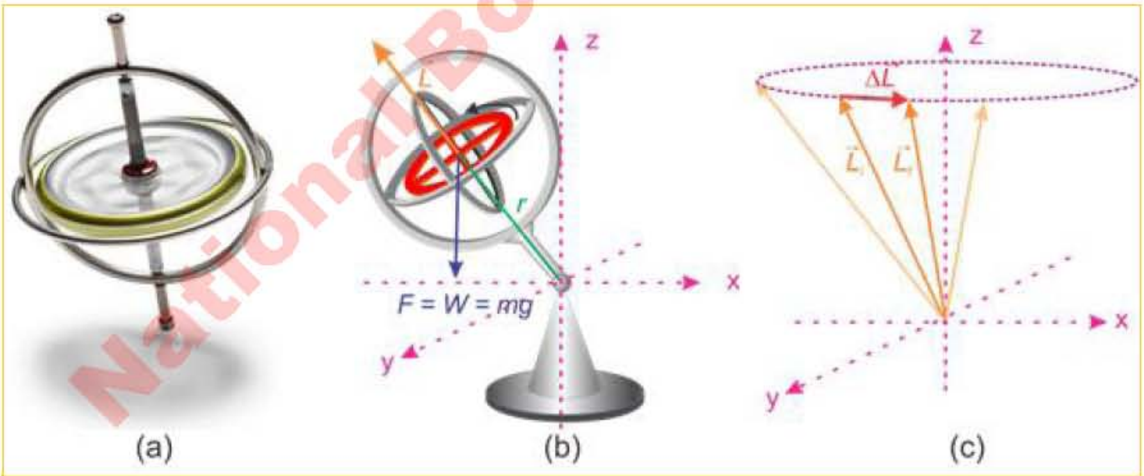


Figure 4.18: Spinning Gyroscope.

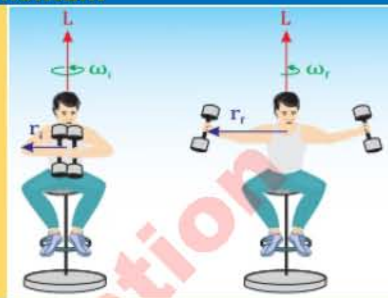
This unusual behaviour can be explained through vector nature of angular momentum, the change in direction of gyroscope will require a torque. The torque is provided by gravitational force as its weight towards the ground. The angular momentum will start to follow the torque as shown in the figure 4.18 (c), the change in angular momentum 'ΔL' is:

$$\Delta L = \tau \times \Delta t$$

Where the torque has the same direction as 'ΔL' and 'Δt' is the duration of time. The same effects can also be observed even if it is lifted by string looped around its lower end.

Activity - Conservation of Angular Momentum

Hold pair of dumbbells in your hand and find a turntable to rotate with full speed by holding dumbbells close to your body. As soon as you extend your arms your rotation speed (angular velocity) will decrease. Again upon drawing your hands nearer towards chest the angular velocity will increase.
Now you know why?



Example 4.3: What is the angular momentum of a 3.6 kg uniform cylindrical grinding wheel of radius 31 cm when rotating at 1150 rpm? (b) How much torque is required to stop it in 7.8 s?

Given: Mass 'm' = 3.6 kg Radius 'R' = 31 cm = 0.31 m
Initial angular velocity 'ω_i' = 1150 rpm = 120.4 rad/s Time duration 'Δt' = 7.8 s

To Find: (a) Angular momentum $L = ?$ (b) Torque $\tau = ?$

Solution: (a) The angular momentum is given as $L = I\omega$

Since moment of inertia for disk is $I = \frac{1}{2}mR^2$, therefore $L = \frac{1}{2}mR^2\omega$

Putting values, we get $L = \frac{1}{2} \times 3.6\,kg \times (0.31m)^2 \times 120.4\,rad/s$ = 20.83 J s

(b) From the relation between torque and angular momentum $\tau = \frac{L_f - L_i}{\Delta t}$

Putting values, where initial angular momentum L_i is 20.83 kg m²/s and final angular momentum L_f is zero (0 kg m²/s).

$$\tau = \frac{(0 - 20.83)\,kg\,m^2/s}{7.8\,s}$$

Therefore, $\tau = -2.67\,kg\,m^2/s^2 = -2.67\,N\,m$

Assignment 4.3

Earth rotates about its own axis. What will be its angular momentum when its average angular speed around its axis is 7.29×10^{-5} rad/s?

4.5 TORQUE AND ANGULAR ACCELERATION

Relationship exists between torque and angular acceleration, just like force and acceleration as in Newton's second law of motion.

4.5.1 For a Point Mass

Consider a mass 'm' rotating at distance 'r' from the axis of rotation as shown in figure 4.19. The force 'F' acting on the mass to rotate it is tangential force. By definition of torque

$$\tau = rF\sin\theta\hat{n} \_\_\_\_\_\_ (1)$$

Since θ = 90° and sin90°=1, magnitude of equation 1 will become: $\tau = rF$ ______ (2)

By Newton's second Law, $F = ma$ ______ (3)

The relation between tangential and angular acceleration is given by:

$$a = r\alpha \_\_\_\_\_\_ (4)$$

Putting equation (4) in equation (3), we get:

$$F = mr\alpha \_\_\_\_\_\_ (5)$$

Putting equation (5) in equation (2), we get

$$\tau = r(mr\alpha) \quad \text{or} \quad \tau = mr^2\alpha \_\_\_\_\_\_ (4.22)$$

Since, the term mr^2 in equation (6) is moment of inertia and can be given from equation (4.16), therefore,

$$\tau = I\alpha \_\_\_\_\_\_ (4.23)$$

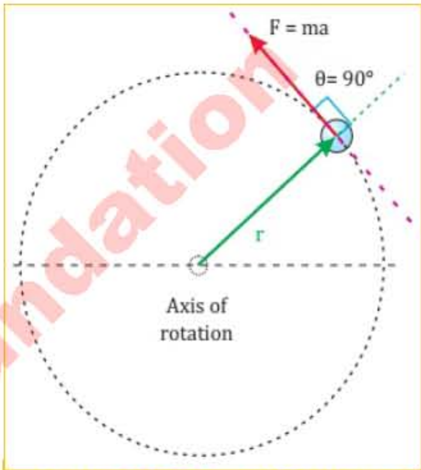


Figure 4.19: Mass 'm' rotating at distance 'r' from axis of rotation.

Equations (4.23) states that torque is moment of inertia times angular acceleration. This statement is similar to Newton's second law of motion F = ma, which gives force as equal to inertia (mass) times acceleration.

4.6 WEIGHTLESSNESS IN SATELLITES

Astronauts in space stations are not in gravity free environment high above the earth. 250 miles out in space, where most space stations orbit, the gravitational field is still quite strong there roughly 95% of what it is on surface of earth. Weightlessness can be created in two ways. One way is to travel millions of miles from gravitational force of large object, where the gravitational force reduces to nearly zero. Or the second and much more practical is to create weightless environment through act of free fall. The space stations are in constant free fall. The satellites orbit the earth at right speed and at right altitude. Inside the space station the astronaut are also falling free, so they appear to float as shown in figure 4.20, and physicists call it weightlessness.

Weightlessness can be enjoyed in amusement parks momentarily, however living life in space station is not easy, besides the dangers of space travel and time away from family in isolation, astronauts feel many health issues related to micro gravity. Their bones and muscles get weakened, cardiovascular system is affected and immune system is weakened. Apart from all these health issues some everyday things are near-impossible for astronauts to do in space. Their basic eating, sleeping, and showering habits are modified. They even can't cry, they face difficulty in digesting food and even in urination and excretion. Rotational simulated gravity has been proposed as a solution in human spaceflight to the adverse health effects caused by prolonged weightlessness.

Figure 4.20: Weightlessness in satellites.

4.7 ARTIFICIAL GRAVITY

The gravity produced artificially in the satellites to get rid of weightlessness is called artificial gravity.

It can be created by rotating the space-station around its own axis as shown in figure 4.21. The surface of the rotating space station pushes on an object with which it is in contact and thereby provides the centripetal force that keeps the object moving on a circular path. In space stations the astronauts feel weightless and cannot work effectively. In order to overcome this difficulty artificial gravity can be provided by rotating it about its own axis.

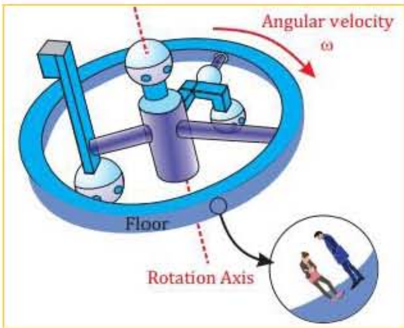


Figure 4.21: Artificial gravity.

To describe artificial gravity, consider a circular tube shaped part of the space station in which artificial gravity will be provided to the occupants of the space station. Let it have the radius 'R' and rotate with velocity 'v' as shown in the figure 4.21. The centripetal acceleration experienced at any point on the outer rim is

$$a_c = \frac{v^2}{R} \quad \text{\_\_\_\_\_\_} \quad (1)$$

Linear Velocity: From equation (1), we know that: $v^2 = a_c R$

Therefore, $v = \sqrt{a_c R}$

To provide the same force as the force of gravity this centripetal acceleration, and hence centripetal force, must be equal to the acceleration due to gravity i.e. $a_c = g$

Hence $v = \sqrt{gR}$ ______ (4.24)

Angular Velocity of Satellite: Since the relation between linear and angular velocity is

$$v = \omega R \text{ ______ } (2)$$

Comparing equations (2) and (4.24), we get:

$$\omega R = \sqrt{gR} \qquad \text{Or} \qquad \omega = \frac{\sqrt{gR}}{R} = \sqrt{\frac{gR}{R^2}}$$

Therefore, $$\omega = \sqrt{\frac{g}{R}} \text{ ______ } (4.25)$$

Time Period of Satellite: Time period is the time required for the satellite to complete one rotation is

$$T = \frac{2\pi R}{v} \text{ ______ } (3)$$

Since $v = R\omega$, therefore

$$T = \frac{2\pi R}{\omega R} \qquad \text{Or} \qquad T = \frac{2\pi}{\omega} \text{ ______ } (4)$$

Putting the values from equation (4.25) in equation (4), we get:

$$T = 2\pi\sqrt{\frac{R}{g}} \text{ ______ } (4.26)$$

Frequency of Satellite: Since frequency is the reciprocal of time period, i.e., $f = \frac{1}{T}$, so, from equation (4.26) we get:

$$f = \frac{1}{2\pi}\sqrt{\frac{g}{R}} \text{ ______ } (4.27)$$

EXAMPLE 4.4: An 80.0 kg astronaut stands in the rim of rotating ring shaped space station providing him sufficient artificial gravity g = 9.8 m/s2. If the radius of the space station is 1.5 km. Calculate his (a) angular velocity, (b) time period and (c) frequency of rotation.

Given: mass of astronaut = m = 80.0 kg radius of space ship = R = 1.50 km = 1500 m

To Find: (a) angular velocity 'ω' =? (b) time period 'T' = ? (c) frequency 'f' = ?

SOLUTION: (a) The angular velocity for artificial satellite is $\omega = \sqrt{\frac{g}{R}}$

Putting values, we get: $\omega = \sqrt{\frac{9.8\,m/s^2}{1500\,m}}$ or $\omega = 0.08\,rad/s$

(b) The time period for artificial satellite is $T = 2\pi\sqrt{\frac{R}{g}}$

Putting values, we get: $T = 2 \times 3.14\sqrt{\frac{1500\,m}{9.8\,m/s^{2}}}$ or $T = 77.73$

(c) Since frequency is the reciprocal of time period $f = \frac{1}{T}$

Putting values, we get: $f = \frac{1}{77.73\,s}$ or $f = 0.013\,Hz$

Assignment 4.4

A space ship, having cylindrical shape, is rotated at a speed of 20 rpm about its axis in order to provide artificial gravity to its inhabitants. If the spaceship has a diameter of 8 m, find the artificial gravity it provides.

SUMMARY

- **Angular velocity:** The rate of change of the angle with which an object moves on a circular path.
- **Tangential acceleration:** The acceleration in a direction tangent to the circle at the point of interest in circular motion.
- **Angular acceleration:** The rate of change of angular velocity with time
- **Centripetal acceleration:** The acceleration of an object moving in a circle, directed toward the center.
- **Centripetal force:** Any net force causing uniform circular motion.
- **Moment of inertia:** Mass times the square of perpendicular distance from the rotation axis; for a point mass, it is $I = mr^2$ and, because any object can be built up from a collection of point masses, this relationship is the basis for all other moments of inertia
- **Torque:** The turning effectiveness of a force and is defined as product of moment of inertia and angular acceleration ($\tau = I\alpha$).
- **Angular momentum:** The product of moment of inertia and angular velocity ($L = I\omega$)
- **Torque** is the time rate of change of angular momentum. $\tau = \Delta L/\Delta t$
- Angular momentum is conserved, i.e., the initial angular momentum is equal to the final angular momentum when no external torque is applied to the system.

EXERCISE

Multiple Choice Questions

Encircle the Correct option.

1) Which of the following is NOT correct?

A. $v = r\omega$ B. $\alpha = v \times r$ C. $a = r\alpha$ D. $\alpha = \frac{\Delta\omega}{\Delta t}$

2) A car turns around a curve at 30 km/h. If it turns at double the speed, the tendency to overturn is

A. doubled B. quadrupled C. halved D. unchanged

3) Moment of inertia of a spinning body, about a certain axis, doesn't depend on
A. distribution of mass around the axis B. orientation of the axis
C. mass of the body D. angular velocity of the body

4) The change in angular momentum of a rod, when a torque of 2.5 Nm is acted upon it for 2 seconds, is
A. 1.25 Js B. 2.5 Js C. 5 Js D. ZERO

5) If size (length) of the wings of a fan is increased, its rotational speed, for the same voltage and current, will
A. increase B. decrease C. may increase or decrease D. remain constant

6) In a body, angular acceleration is produced by
A. net force B. power C. pressure D. net torque

7) An astronaut feels weightless inside the International Space Station. It is because the International Space Station is
A. outside the gravitational field of earth B. freely falling
C. at rest D. in motion

Short Questions

Give short answers of the following questions.

1) What is the value of angular acceleration of the minute hand of your wrist watch?
2) Define the following angular quantities: Displacement, velocity, acceleration.
3) Determine the relation between (a) linear and angular displacement. (b) linear and angular velocity (c) linear and angular acceleration.
4) Is centripetal force a fundamental force or a force provided by any of the fundamental forces? Can any combination of the fundamental forces provide centripetal force?
5) There are generally double tyres in heavy vehicles on one side of an axle. Will its moment of inertia be different from that of a single tyre?
6) Why it is best to have the blades rotate in opposite directions for a helicopter having two sets of lifting blades?
7) If diameter of earth becomes half and there is no change in its mass, what affect will be there on the rotational speed of earth around its own axis?
8) Why does in circular motion, a tangential acceleration can change the magnitude of the velocity but not its direction?
9) Why does usually the value of artificial gravity is smaller than 9.8 m/s^2?
10) Why is gyroscope used in aeroplanes?
11) How the rotation of a flywheel helps to even out the power delivery from the engine?

Comprehensive Questions

Answer the following questions in detail.

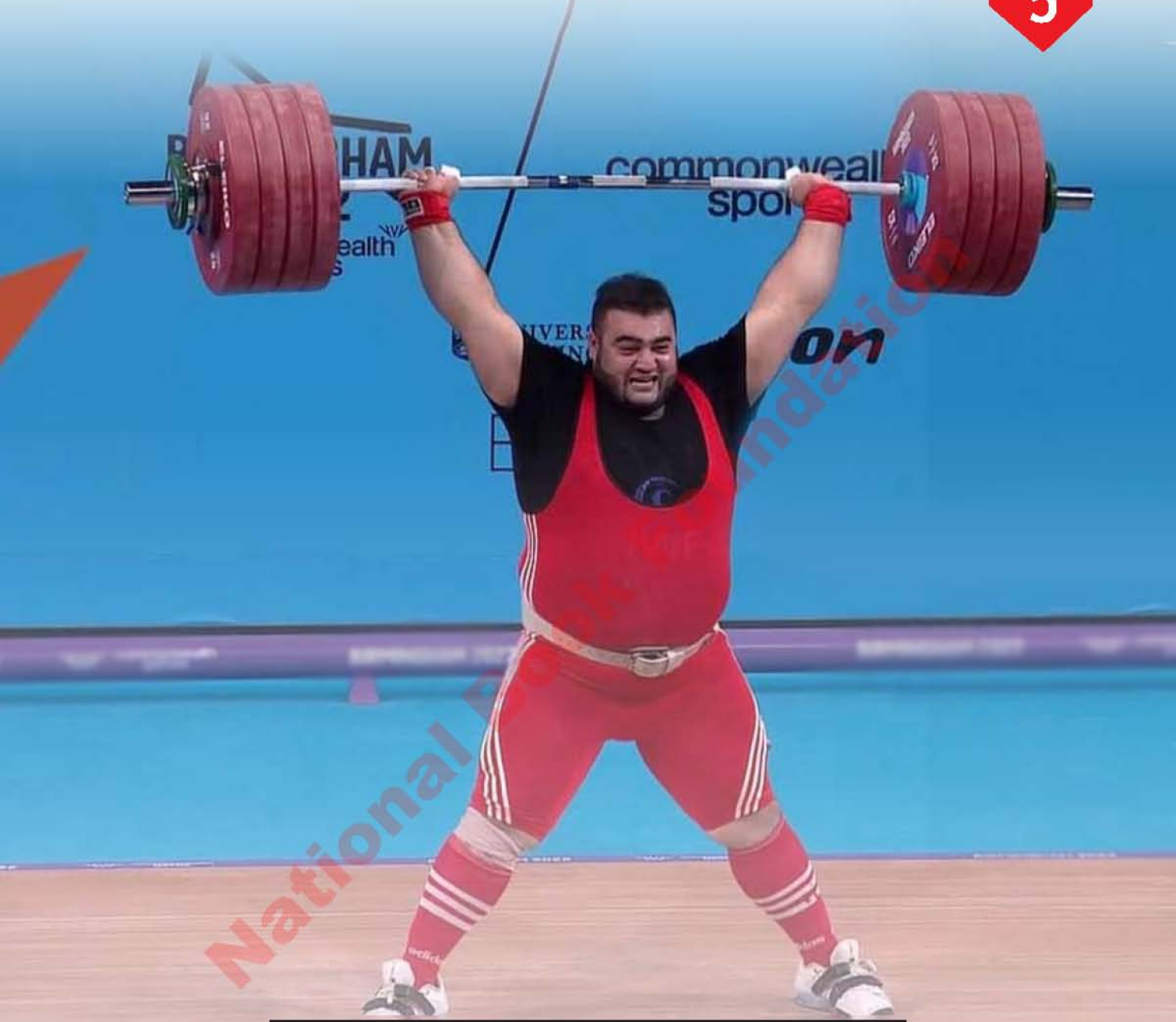
1) What is centripetal force? Explain. Write down at least two applications where centripetal force plays its vital role.
2) What is moment of inertia? Derive its relation for rigid body.
3) Define and derive expression for angular momentum of a body. Also deduce the relation between angular momentum and torque.
4) Explain conservation of angular momentum using practical life examples.
5) Derive the relation between torque and angular acceleration.
6) Why do astronauts feel weightless in a satellite? What is meant by artificial gravity? How can artificial gravity be produced in a satellite?

Numerical Problems

1) What will be the angular velocity of fly wheel of an engine if it travels 3000 revolutions in a minute? **(Ans. 314 rad/s)**
2) A car is passing through a turn that is in the form of an arc of a circle of radius 14.5 m. What will be the maximum speed limit (the speed at which the car can cross the bridge without losing contact with the road) if the centre of gravity of the car is 0.5 m from the ground? **(Ans. 12.1 m/s)**
3) A PT teacher rotates his stick at the axis that passes through its centre. If mass of the stick is 200 g and its length is 0.8 m, find its moment of inertia? **(Ans. 0.01 kg m^2)**
4) A football of mass 450 g rotates with an angular speed of 10 revolution/s. If its radius is 11 cm, compute its angular momentum? **(Ans. 0.036 Js)**
5) A merryman in a circus is standing with his arms extended on a turn table rotating with angular velocity 10 rad/s. He brings his arms closer to his body so that his moment of inertia is reduced to one third of the initial value. Find his new angular velocity. **(Ans. 30 rad/s)**
6) A boy exerts a force of 200 N at the edge of the 30.0 kg merry-go-round, which has a 2.0 m radius. Calculate the angular acceleration produced (a) when no one is on the merry-go-round and (b) when the boy having 20.0 kg weight sits 1.5 m away from the center. (ignore friction). **(Ans. 6.67 rad/s^2, 3.8 rad/s^2)**
7) A wheel shape space station provides an artificial gravity of 5.00 m/s^2 to its inhabitants. If it has a diameter of 100 m, find its angular speed in rpm. **(Ans. 3 rpm)**

UNIT 5

WORK AND ENERGY



Student Learning Outcomes (SLOs)

The students will:

- Derive the formula for kinetic energy [Using the equations of motion].
- Deduce the work done from force-displacement graph.
- Differentiate between conservative and non-conservative forces.
- Utilize the work - energy theorem in a resistive medium to solve problems.

In the sport of weightlifting, the task is to pick up a very large mass, lift it over our head, and hold it there at rest for a moment. This action is an example of doing work by lifting and lowering a mass. Weightlifting requires energy from the body's metabolic processes, specifically from the breakdown of a molecule called adenosine triphosphate (ATP). Additionally, weightlifting can also utilize stored glycogen in muscle tissue, which can be broken down into glucose to provide energy.

5.1 WORK

The work done on an object is the scalar (or dot) product of force F and displacement d.

$$W = \vec{F} \cdot \vec{d}$$

Or $$W = F\, d \cos\theta \quad (5.1)$$

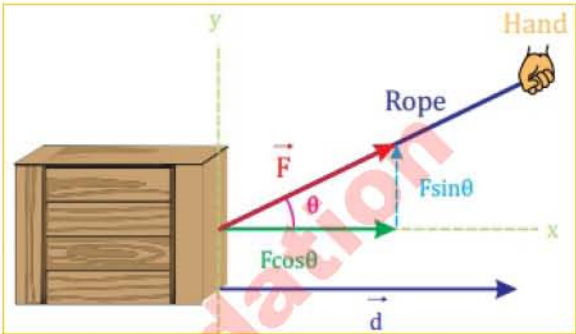


Figure 5.1: Work done on an object when a force 'F' produces displacement d.

In equation (5.1), 'F' is the magnitude of force, 'd' is the magnitude of displacement, and 'θ' is the angle between force and the displacement. From the figure 5.1, we can see that the force can be resolved into two components $F\cos\theta$ and $F\sin\theta$. Here, $F\cos\theta$ is the component of force along the direction of displacement which is the effective component. Whereas, $F\sin\theta$ is the component of force which is perpendicular to the direction of displacement and therefore plays no role in doing work.

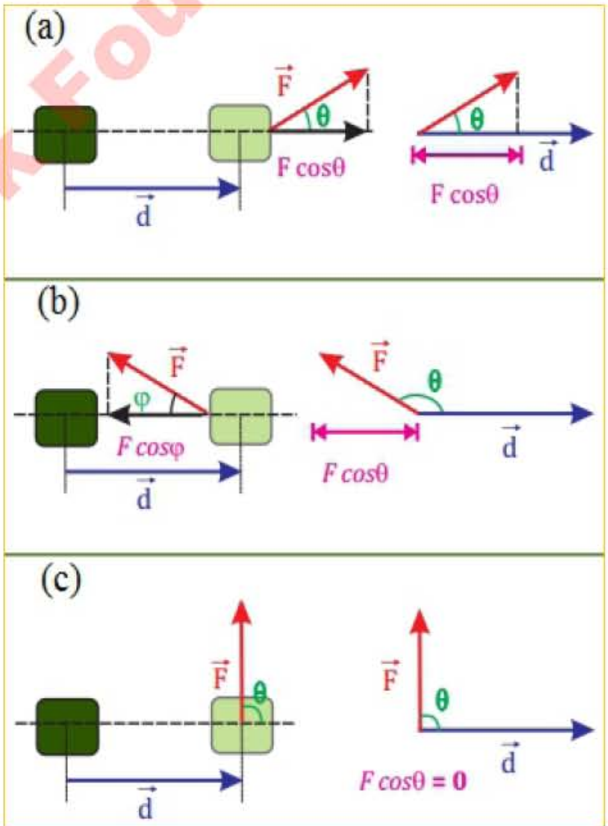


Figure 5.2: (a) positive work (b) negative work (c) zero work.

Units of Work: The SI unit of work is the joule (J) (named in honor of the 19th-century English physicist James Prescott Joule). From above equation we see that the unit of work is the unit of force multiplied by the unit of distance. In SI the unit of force is the newton and the unit of distance is the metre, so 1 joule is equivalent to 1 newton-metre (N m).

$$1\ J = 1\ N\ m$$

Dependence of Work: The work done depends upon force 'F', displacement 'd' and the angle 'θ' between them as shown in figure 5.2.

a) Positive Work: When the force has a component in the same direction as the displacement ($0° \le \theta < 90°$), $\cos\theta$ in above equation is positive and the work W is positive.

b) Negative Work: When the force has a component opposite to the displacement ($90^o < \theta \le 180^o$), *cos* θ is negative and work done by the force will be negative.

c) Zero Work: When the force is perpendicular to the displacement, $\theta = 90^o$ then the work done by the force is zero.

Work Done from Force-Displacement Graph: The area under the force-displacement graph gives the work done.

A. Work Done by Constant Force: In figure 5.3, graph for a constant force to produce net displacement is shown, here the blue shaded area of rectangle represent work done by the constant force.

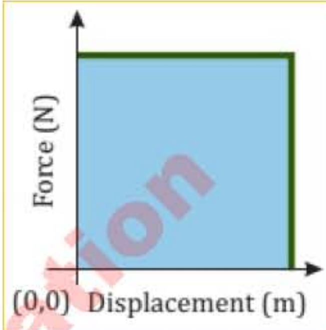


Figure 5.3: Work done by constant force.

B. Work Done by Variable Force: In figure 5.4, graph with increasing force to produce net displacement is shown, here the blue shaded area of triangle represent work done by the increasing force. However, in many situations of daily life, the force is variable in many different ways, and therefore we get different graphs for area under the curve for work done by such forces.

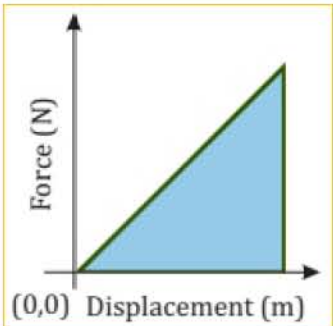


Figure 5.4: Work done by variable force.

For example, when we stretch a spring, the more we stretch it, the harder we have to pull, so the force we exert is not constant as the spring is stretched. When a rocket moves away from earth the work is done against the force of gravity which varies as inverse of the square of distance from the center of earth.

Consider a graph in figure 5.5, is drawn between *Fcosθ* and d. To find the total work done, we divide the displacement into number of small displacements Δd_1, Δd_2, Δd_3, ..., Δd_n, with corresponding effective component of forces are $F_1 \cos\theta_1$, $F_2 \cos\theta_2$, $F_3 \cos\theta_3$, ..., $F_n \cos\theta_n$.

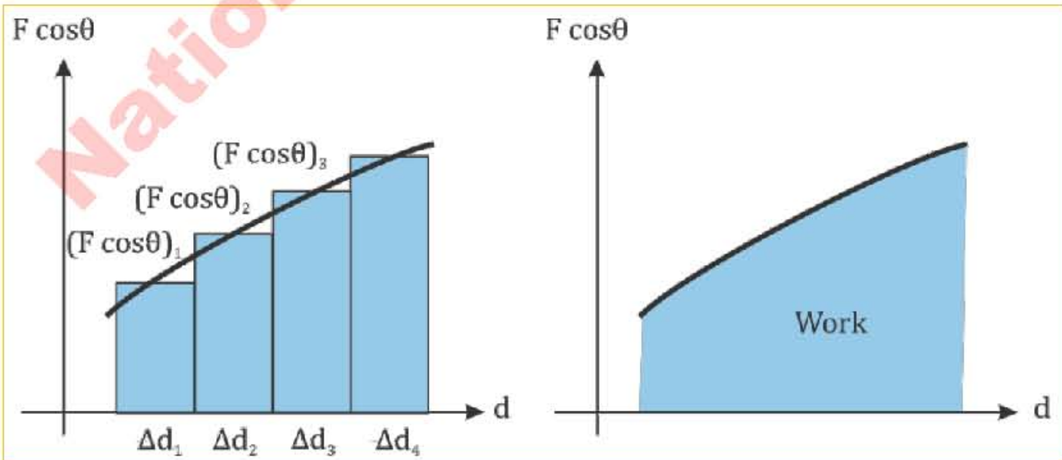


Figure 5.5: Graphical analysis of work done by variable force.

The total work done will now be the sum of all the individual work done.

$$W_{Total} = W_1 + W_2 + W_3 + ... + W_n$$

As, $W_1 = F_1 \cos\theta_1 \Delta d_1$, $W_2 = F_2 \cos\theta_2 \Delta d_2$,

$W_3 = F_3 \cos\theta_3 \Delta d_3$, and $W_n = F_n \cos\theta_n \Delta d_n$.

So, the total work done is:

$$W_{Total} = F_1 \cos\theta_1 \Delta d_1 + F_2 \cos\theta_2 \Delta d_2 + F_3 \cos\theta_3 \Delta d_3 + ... + F_n \cos\theta_n \Delta d_n \quad \text{\_\_\_\_\_ (5.2)}$$

In compact form the above equation can be written as:

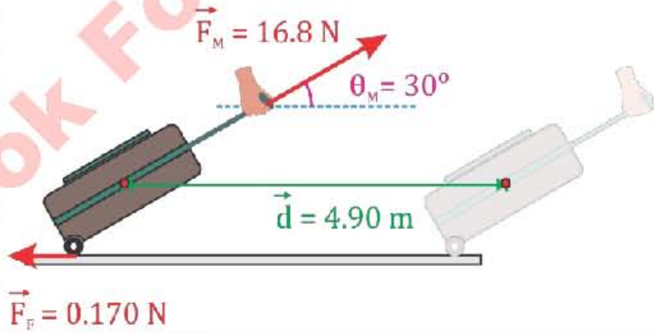
$$W_{Total} = \sum_{i=1}^{n} F_i \cos\theta_i \Delta d_i \quad \text{—— (5.3)}$$

James Prescott Joule (1818 – 1889)

Joule showed that heat was not a substance but, instead, the transfer of energy. He found that thermal energy produced by stirring water or mercury is proportional to the amount of energy transferred in the stirring.

Thus, the work done by a variable force is equal to the area under the *Fcos θ* and *d* curve.

EXAMPLE 5.1: Mehwish is dragging a suitcase through an airport and pulls it a distance of 4.90 m along level ground. She is applying a constant force of 16.8 N in 30° with the horizontal. A 0.170 N friction force opposes the suitcase's motion. Find the work done by (a) Mehwish (b) frictional force and (c) net work done by all the forces acting on the suitcase.



Given: Force by Mehwish 'F_M' = 16.8 N Angle 'θ_M' = 30°

Force of friction 'F_f' = 0.170 N Angle 'θ_F' = 180° Displacement 'd' = 4.90 m

To Find: (a) Work by Mehwish W_M = ? (b) Work by friction W_f = ?

(c) Net work done W_{net} = ?

Solution: (a) The work done by Mehwish 'W_M' can be calculated as:

$$W_M = \vec{F}_M \bullet \vec{d} \quad \text{or} \quad W_M = F_M\, d \cos\theta_M$$

Putting values: $W_M = 16.8\,N \times 4.90\,m \times \cos 30^\circ$ or $W_M = 71.3\,Nm$

Hence, $W_M = 71.3\,J$

(b) The work done by friction 'W_f' can be calculated as:

$$W_f = \vec{F}_f \bullet \vec{d} \quad \text{or} \quad W_f = F_f\, d \cos\theta_f$$

Putting values: $W_f = 0.170\,N \times 4.90\,m \times \cos 180^\circ$ or $W_f = -0.833\,J$

(c) Although there is force of gravity due to weight of the suitcase acting on it. But as it is perpendicular to the direction of motion, it does not have an effective component in work. The net work done 'W_{net}' is the sum of work done by Mehwish 'W_M' and friction 'W_f'.

$$W_{Total} = W_M + W_f$$

Putting values: $$W_{Total} = 71.3\,J + (-0.833\,J)$$

Therefore, $$W_{Total} = 70.467\,J = 70.5\,J$$

Assignment5.1

A box having 40 kg mass is dragged on a frictionless inclined surface to a height of 8 m. If the inclined plank makes an angle of 20° with the earth, find the work done against gravity.

5.2 CONERVATIVE & NON-CONSERVATIVE FIELDS

The region around a body where it can influence other bodies by a force associated with it is called field of force or simply force field. Just like electric field, viscous field and gravitational field.

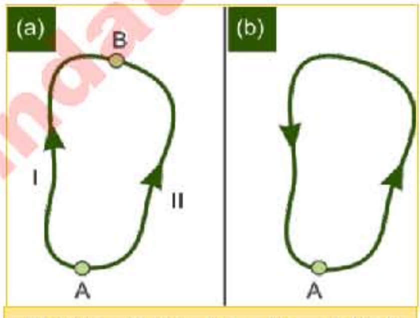


Figure 5.6: (a) An object moves between points A and B via two different paths 1 and 2. (b) The object makes a round trip back to A.

5.2.1 Conservative Field

A field is said to be conservative if it has two important properties:

1. Work Done is Independent of the Path Taken: If the work done on a particle moving between any two points A and B is same for path I and II as shown in figure 5.6 (a), the field will be conservative.

2. Work Done around a Closed Path is Zero: If the work done on a particle moving through any closed path (the path in which the beginning and end points are same) gives zero as shown in figure 5.6 (b), such a field will be conservative.

Gravitational and electric fields are examples of conservative fields and the associated forces are conservative forces.

5.2.2 Non-Conservative Field

A non-conservative field is that field in which work done depends upon the path followed or the work done along a closed path is not zero.

Frictional field is a non-conservative field, and frictional or drag forces are non-conservative forces because when an object is moved in frictional field, the work done against frictional force depends upon the path followed.

Frictional force, viscous drag and air resistance are all examples of non-conservative forces and the fields where they act are termed as non-conservative fields.

5.3 KINETIC ENERGY

The energy possessed by a body due to its motion is called Kinetic energy.

A boy kicks a football; it moves because it possess Kinetic energy. Now think a tennis ball and a football moving with same speed. Which possess greater ability to do work? Of course, the football with larger mass, which is difficult to stop. Similarly, now two footballs are approaching you with different speeds, which can do more work? Again, the football with greater speed is difficult to stop. Thus, the object's mass and its speed contribute to its Kinetic energy. Like all energies, Kinetic energy is also a scalar quantity.

Consider a situation in which all the work done transfers only kinetic energy to a cricket ball. Let the cricket ball is initially at rest $v_i = 0$ m/s, and a horizontal force F is applied to move it through a displacement 'S' and achieve a final velocity $v_f = v$ as shown in figure 5.7. This work done W appears as the kinetic energy E_k. Such that:

$$W = E_K \quad FS \_\_\_\_\_\_ (1)$$

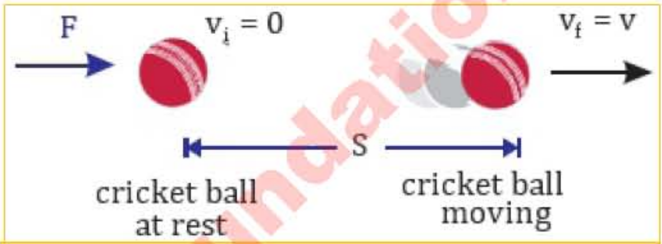


Figure 5.7

By Newton's Second Law of motion

$$F = ma \_\_\_\_\_\_ (2)$$

From third equation of motion, distance S can be written as:

$$S = \frac{v_f^2 - v_i^2}{2a} \_\_\_\_\_\_ (3)$$

Putting equations (2) and (3) in equation (1), we get:

$$E_K = \frac{1}{2}m\left(v_f^2 - v_i^2\right) \_\_\_\_\_\_ (4)$$

As the object (cricket ball) started from rest therefore $v_i = 0$ and $v_f = v$.

Therefore, $$E_K = \frac{1}{2}mv^2 \_\_\_\_\_\_ (5.4)$$

Although derived for cricket ball, the equation (5.4) in general shows the relation for the kinetic energy of any moving object. For example, an iron ball of 1.0 kg moving with a speed of 2.0 m/s has a Kinetic energy of 2 J.

Equally important, it demonstrate the work kinetic energy theorem which states that work done on an object is equal to change in energy i.e. W = Δ E, where 'W' is the work done and ΔE is the change in energy.

5.4 WORK - ENERGY PRINCIPLE

Since work is defined as the movement of an object through a distance, energy can also be described as the ability to move an object through a distance. The net work done on an object is equal to the change in the object's kinetic energy, i.e.,

$$W_{net} = K.E_f - K.E_i = \Delta K.E \_\_\_\_ (5.5)$$

Where, the change in the kinetic energy is due to the object's change in speed.

Work-Energy Theorem is also valid for potential energy. However, potential energy cannot be defined only for conservative forces.

In situations where potential energy can be defined, change in potential energy is exactly equal to the **negative** of change in kinetic energy, in which case the Work-Energy theorem becomes

$$W_{net} = \Delta K = -\Delta U \quad \text{\_\_\_\_\_\_} \quad (5.6)$$

5.4.1 Work - Energy Theorem in Resistive Medium

Why a skydiver is quite confident having a parachute? Probably she relies on resistive forces balance the motion. A resistive force on a moving object opposes the motion of the object, or prevent a stationary object from moving, for example friction, viscous force etc.

When forces are acting on an object there are energy transformations occurring, which means that work will be done. The work done '*W*' on an object by an applied force 'F' is the sum of the gain in kinetic energy '*ΔK.E*' of the object and the work done by the object against the resistive force 'W_r'. That is,

$$W = \Delta K.E + W_r$$

The work done by the object against the resistive force takes energy away from the object, decreasing its kinetic energy. Mathematically

$$W_r = W - \Delta K.E \quad \text{\_\_\_\_\_} \quad (5.7)$$

Both Newton's laws and work energy theorem can be used to solve problems, however if the forces are not constant Newton laws will prove difficult to apply.

5.4.2 Implications of Energy Loses in Practical Devices and Efficiency

Input of Mechanical Machine: The energy supplied to a mechanical machine is called input. Input is equal to the product of effort 'p' and the distance 'd' through which effort acts.

$$input = p \times d \; ... \; (1)$$

Output of Mechanical Machine: The work done by a mechanical machine is called output. Output is the product of load 'W' and the distance 'D' through which the load lifts.

$$output = W \times D \; ... \; (2)$$

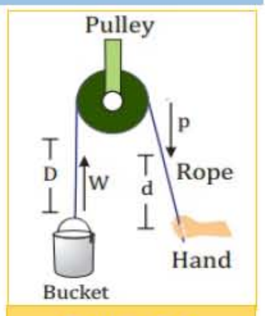


Figure 5.8:

Efficiency: The efficiency is the ratio of work output to the work input, and can be expressed in percentage as:

$$efficiency = \frac{useful\ energy\ output}{energy\ input}$$

Efficiency is the ratio of similar quantities and therefore has no unit.

The efficiency cannot be greater than 1 (or 100%), in fact it cannot be equal to 1 (or 100%) for real machines. In case of pulley system, the efficiency is only 40%, rest of the 60% goes into waste and converted into unwanted forms of energy.

Ideal Machine: For an ideal machine only, the input will be equal to output, this means that no energy is wasted and all energy is converted into useful work. Therefore, the efficiency of an ideal machine is 100 %.

$$output = input \qquad or \qquad efficiency = 1$$

Almost all energy transformation technologies operate at efficiencies less than 100%, most of the wasted energy becomes thermal energy.

For example, automobiles are highly inefficient. Suppose that an amount of fuel containing 1000 J of chemical potential energy is used by an automobile's engine. Only 10 percent of the energy is available to do work.

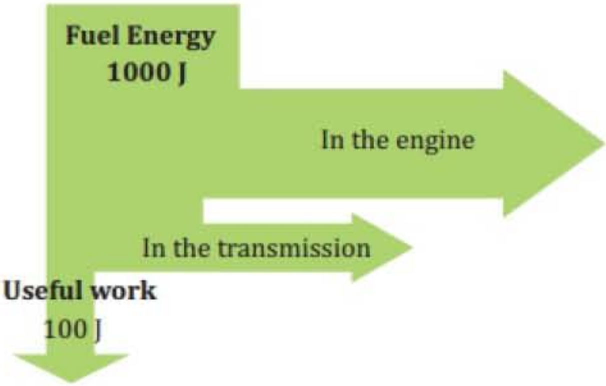


Table 5.1: Typical efficiencies of energy transformation technologies.

Device	Efficiency (%)
electric heater	100
electric generator	98
hydroelectric power plant	95
large electric motor	95
home gas furnace	85
wind generator	55
fossil fuel power plant	40
automobile engine	25
fluorescent light	20
incandescent light	5

$$\text{efficiency} = \frac{\text{energy output}}{\text{energy input}} \times 100$$

or

$$\text{efficiency} = \frac{100\,J}{1000\,J} \times 100 = 10\ \%$$

Only about 10% (or 100 J) of useful work is done in producing the kinetic energy of the moving car, rest of the energy is wasted in engine and cars transmission. Some other examples of energy losses in practical devices are given in table 5.1.

EXAMPLE 5.2: In a science experiment a 3.00 kg water rocket, is launched from ground. The rocket's total energy at the top of its flight is 2352 J. (a) What was the rocket's launching speed? (b) What height did the rocket went? (c) What is the kinetic energy and potential energy of the rocket 2.5 s after its launch?

Given: Mass of rocket 'm' = 3.00 kg   Total energy 'E_T' = 2352 J

Acceleration due to gravity 'g' = 9.8 m/s^2 Time 't' = 2.5 s

To Find: (a) Initial speed 'v' = ?   (b) Height 'h' = ?

(c) 'K.E' of rocket after 2 s = ?   'P.E' of rocket after 2 s = ?

Solution: (a) By law of conservation of energy, when the rocket is launched it K.E is converted into P.E, and at height 'h' this potential energy is equal to the total energy.

$$K.E = E_T \quad \text{or} \quad \frac{1}{2}mv^2 = E_T \quad \text{or} \quad v^2 = \frac{2 \times E_T}{m}$$

taking square root on both sides:

$$v = \sqrt{\frac{2 \times E_T}{m}}$$

putting values:

$$v = \sqrt{\frac{2 \times 2352\,J}{3.00\,kg}} \quad \text{or} \quad v = 39.6\,m s^{-1}$$

(b) At height 'h', the potential energy is equal to the total energy.

$$P.E = E_T \quad \text{or} \quad mgh = E_T \quad \text{or} \quad h = \frac{E_T}{mg}$$

Putting values: $h = \dfrac{2352J}{3.00kg \times 9.80ms^{-2}}$ or $h = 80m$

(c) To find 'K.E' after 2.5 seconds we will first have to calculate the speed of rocket by using first equation of motion along y-axis

$$v_f = v_i - g \times t$$

Putting values: $v_f = 39.6ms^{-1} - 9.8ms^{-2} \times 2.5s$

Therefore, $v_f = 15.1ms^{-1}$

Putting this value in the kinetic energy equation $K.E = \frac{1}{2}mv_f^2$

Putting values: $K.E = \frac{1}{2}3.00\ kg \times (15.1ms^{-1})^2$ or $K.E = 342J$

To find the potential energy, we will use law of conservation of energy

$$E_T = K.E + P.E \quad \text{Or} \quad P.E = E_T - K.E$$

putting values: $P.E = 2352J - 342J$

Hence, $P.E = 2010J$

Assignment 5.2

A crane holding 5000 kg of mass at a height of 12 m. Suddenly, the crane un-holds the mass. Find the kinetic energy of the mass just before striking the ground.

SUMMARY

- **Work:** The work done on a body by a constant force is defined as the product of the displacement and the components of the force in the direction of the displacement.
- The energy possessed by a body due to its motion is called **Kinetic energy.**
- **Gravitational field as conservative field:** Gravitational field is conservative field as work done is independent of the path followed and work done along the closed path is zero.
- A **non-conservative field** is that field in which work done depends upon the path followed or the work done along a closed path is not zero.
- **Efficiency:** The efficiency is the ratio of work output to the work input.

EXERCISE

Multiple Choice Questions

Encircle the Correct option.

1) If the unit of force and displacement travelled each be increased five times, then the unit of work will be increased by

A. 25 times B. 10 times C. 5 times D. 0 times

2) The force that acts on a body but does no work is

A. gravitational force B. frictional force C. elastic force D. centripetal force

3) The odd force from the following is

A. gravitational force B. elastic force C. frictional force D. electric force

4) The work done by a body while covering a vertical height of 10 m is 500 J. By how much amount does the energy of the body change?

A. 500 J B. - 500 J C. 50 J D. - 50 J

Short Questions

Give short answers of the following questions.

1) Is it possible that a force is acting on a body and the body is in motion due to this force but the work done after certain time is zero?
2) Some non-conservative forces are acting on a body. Can they change the total mechanical energy of the body?
3) Kinetic energy and work are related. Can kinetic energy ever be negative? Can work ever be negative?
4) Differentiate between conservative and non-conservative forces.
5) What is the work done by the moon as it revolves around the Earth?

Comprehensive Questions

Answer the following questions in detail.

1) Explain work done by a constant and variable force using force-displacement graph.
2) What is work – energy theorem. Explain in detail. Also, write some implications of energy loses in practical devices and efficiency.

Numerical Problems

1) A car carrying truck unloads a 1500 kg car using a plank as shown in figure. If the plank makes an angle of 30^0 with the ground and its upper end is at 2 m height, what will be the work done by gravitational force? Also, draw the force-displacement graph for. **(Ans. 25.46 kJ)**

2) A car, having a total mass of 1500 kg (including the driver), is travelling at a speed of 40 kph through a straight path. How much work will be required to stop the said car if its brakes fail and engine turns off? **(Ans. 92.6 kJ)**

3) A ball of mass 100 g is released from a height of 30 m. If the ball encounters an air resistance of 0.4 N, find the kinetic energy of the ball just before striking the ground. **(Ans. 17.4 N)**

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قومی ترانہ

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تو نشانِ عزمِ عالی شان ارضِ پاکستان
مرکزِ یقین شاد باد!

پاک سر زمین کا نظام قُوّتِ اُخُوّتِ عوام
قوم ، مُلک ، سلطنت پائندہ تابندہ باد!
شاد باد منزلِ مُراد!

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